Rising Earnings Instability, Portfolio Choice, and Housing Prices *

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Abstract

This paper studies the effects of the U.S. rising earnings inequality between the late 1960s and the mid 1990s on the portfolio allocation and prices of assets. In order to investigate the link, a life-cycle general equilibrium model is constructed where distinct characteristics of the housing asset is explicitly modeled and asset prices are determined in equilibrium. It is shown that the model can produce a 9% rise in the housing price, which is about 40% of the changes in the U.S. data (24%). An increased demand for precautionary savings and the general equilibrium effect play a crucial role here. A higher earnings volatility induces a higher demand for financial assets. As the return of financial assets declines due to the general equilibrium effect, the demand for housing assets increases as well. The paper also examines the effects of the rising earnings inequality on the aggregate amount of debt. Interestingly, contrary to the U.S. data, the model predicts a *decline* in the total amount of secured debt. A higher earnings volatility induces a higher amount of debt in complete markets models, but an increased demand for savings for precautionary motive dominates the positive effect to the amount of debt. The model also shows that incorporating housing assets into the model does not make a significant difference in the effect of the rising earnings inequality on the consumption inequality.

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1 Introduction

One of the most striking changes that occurred in the U.S. labor market since the late 1960s is the rising earnings inequality. Using different data sources and different methodologies, it is confirmed that the labor income in the U.S. in the 1990s is substantially dispersed than in the 1960s. Besides, attempts are made to decompose the changes in the earnings inequality into permanent differences in earnings and the volatility of earnings, because the effect of changing earnings inequality on the consumption and welfare inequality crucially depends on the nature of the earnings inequality (Attanasio, and Richard Blundell and Preston (2002)). The literature finds that the both the persistent differences in earnings across different workers and the size of the earnings volatility increased over the period.

So far the main focus of the literature is to study the effect of the rising inequality of wages and earnings on the inequality of consumption and welfare. Krueger and Perri (2003b) and Krueger and Perri (2003a) study the relationship between rising income inequality and consumption inequality, theoretically and empirically. Heathcote, Storesletten and Violante (2003) study implications of the rising wage inequality on the inequality of hours, earnings, and consumption.

The current paper is related to these literature but different in the sense that the link between the rising earnings inequality and savings behavior, instead of consumption behavior is studied. There is a reason that such study is important. As summarized in the next section, the period of rising earnings inequality also observes significant changes in the portfolio allocation and price of assets. Studying the link between the rising earnings inequality and the changes in the portfolio choice and asset prices will reassure the importance of the rising earnings inequality, or suggest other important changes during the same period which also had significant macroeconomic implications.

In order to investigate the effects of the rising earnings inequality on portfolio choice and asset prices, a life-cycle general equilibrium model with the following two key features is constructed: *(i)* the distinct characteristics of housing assets are explicitly modeled, and *(ii)* the prices of housing and financial assets are determined endogenously. The model is carefully calibrated to replicate the macroeconomic characteristics of the U.S. economy. An important part of the calibration process is estimating the time-varying individual earnings process. The macroeconomic effects of the changes in the earnings inequality is investigated by comparing the models with earnings processes estimated for the 1960s and the 1990s.

Using the model, the paper mainly answers the following three questions. First, we ask whether the rising earnings inequality can be a significant cause of the steady rise in housing prices. National average housing prices have been rising steadily at a faster rate than non-housing consumption goods. Since housing is the most important single goods for most of the U.S. households, an affordability of housing is always a major issue. Our experiments reveal that the changes in earnings inequality can be a major cause of the rise in the housing price. This is mainly due to the increased demand for the savings based on the precautionary motive.

Second, we ask whether the increasing balance of the households' debt over the last 30 years is a consequence of the rising earnings inequality. The balance of debt backed by the value of housing , relative to the size of GDP, almost doubled in the last 30 years. Since a higher earnings volatility in the complete markets economy induces an increase in the balance of debt, it is natural to think that the rising earnings inequality is the reason for the increase of the debt. Surprisingly, our experiment gives a negative answer to the question. Although a higher earnings volatility induces a higher amount of debt, stronger saving incentive turns out to dominate the first effect.

Lastly, we ask whether explicitly considering the housing asset affects the link between the earnings inequality and the consumption inequality. Our experiments indicate that, models with and without housing assets give a very similar effect of the rising earnings inequality on the consumption inequality. As long as households can borrow using their housing assets as collateral, and the consumption of the housing services is not a big item compared with the consumption of non-housing goods, the effect of incorporating housing assets explicitly on the consumption inequality is shown to be limited.

There are several studies which address the upward trend of housing prices. In their well-known work, Mankiw and Weil (1989) argue that the long-run upward trend of housing prices is due to the aging of the baby boomers. Since life-cycle housing holding exhibits a hump shape, a mere increase of the proportion of the households in their prime saving age can increase the aggregate demand for housing asset. Geanakoplos, Magill and Quinzii (2002) show the effect of demographic structure on stock prices. Poterba (2001) discusses empirical support of such effect. The current paper complements the literature studying the source of the upward trend in the housing prices, by exploring the new source, namely the widening earnings inequality.

The current paper is related to the studies that try to measure the proportion of savings that can be accounted for by precautionary motive. Caballero (1991), and more recently Gourinchas and Parker (2002) both claim that around 60% of total savings can be accounted for by precautionary purpose savings. The current paper is related to these papers, in the sense that the current paper implies that the changes in the incentives for precautionary savings can have a significant macroeconomic effect.

There is a growing interest in studying the role of the housing asset in various questions of macroeconomics. Fernández-Villaverde and Krueger (2001) uses a model with durable and financial assets to study empirically and theoretically the life-cycle profiles of durable and non-durable consumption. Díaz and Luengo-Prado (2003) show that the general equilibrium model with uninsured idiosyncratic uncertainty and financial and durable assets can replicate the aggregate composition of wealth in the U.S. economy.

Apart from the features related to the housing asset, the model is a standard general equilibrium life-cycle model with incomplete markets. The seminal works in this class of models are Huggett (1996), and Ríos-Rull (1996). Aiyagari (1994) also develops the incomplete market model but with infinitely-lived agents.

The rest of the paper is organized as follows. Section 2 summarizes the changes in the U.S. portfolio choice and asset prices since the late 1960s. In Section 3, I present the environment of the model and define the equilibrium. The overview of the experiments that will be implemented using the model is shown at the end of this section. Section 4 discusses calibration of the model. Section 5 discusses the computational issues. Section 6 shows the properties of the baseline model economy, calibrated to the 1967 U.S. economy. Section 7 shows the main results of the paper, namely the effects of the rising earnings inequality on housing markets. Section 8 concludes. Appendix A gives the details associated with the estimation of the time-varying earnings process. Appendix B discusses details of the calibration. Finally, Appendix C gives details of the computation.

2 Changes in the U.S. Portfolio Choice and Asset Prices

The section exhibits the changes in the U.S. housing prices and the aggregate amount of debt since the late 1960s. These are the statistics that we ask the model can replicate in response to the changes in the earnings inequality.

2.1 Housing Prices

Figure 1 shows three kinds of housing price indexes, and it's easy to see that all of the three indexes show positive trend since the late 1960s up to now. All indexes are deflated by the CPI of all goods less shelter (including rents for primary residences and imputed rents of owners' housings). The only index starts from 1967, CPI shelter, is normalized to 100 in 1967. The other indexes are normalized such that the index in the first year is equal to the CPI shelter in the corresponding year.

The first index, CPI less shelter, is compiled by the Bureau of Labor Statistics and as a part of Consumer Price Index. This index is constructed by combining the average rent of primary residence and imputed owners' equivalent rent of primary residences. This is an index for the price of housing services, not the housing itself, but is a good proxy for the housing price. Obviously there is an upward trend for the index. The CPI housing price exhibits a 37% increase between 1967 to 1996, and a 49% increase between 1967 to 2004.

The second index, which is called Conventional Mortgage House Price Index (CMHPI) and compiled by Freddie Mac, is a price index for single unit residential housings which are financed by conventional mortgages. Even though there are humps and bumps for the index, the positive trend since the 1970s is obvious. The CMHPI increases by 15% between 1970 and 1996, and 65% between 1970 to 2004.

The third index, which is compiled by Office of Federal Housing Enterprise Oversight (OFHEO) and is called OFHEO House Price Index (OFHEO HPI) uses almost the same data source as CMHPI and thus the trends of the two indexes look very similar. The index starts from 1975.[1] The index increased by 7% between 1974 and 1996, and by 58% between 1974 to 2004.

[1]The value for the first quarter of 1975 is used as the value for 1974.

As for the number representing the rise in the housing price between 1967 and 1996, the index which combines CPI shelter for 1967-1970 and CMHPI for 1970-1996 and is the one in figure 1 is used. The combined index increased by 24% between 1970 and 1996, and 78% between 1967 to 2004.

2.2 Aggregate Amount of Debt

Figure 2 shows the changes in the total amount of debt over GDP in the U.S. during 1967-2002. It is clear that the total amount of debt increased rapidly, from around 50% of GDP in 1980 to around 70% of GDP in 1996, and more than 80% of GDP in 2002. If we decompose the total debt into mortgage loans and consumer credits, both showed an increase over the period but the increase of the mortgage loans is more striking. The total amount of mortgage debt relative to GDP increased from 29% of GDP in 1967 to 46% of GDP in 1996, a 17 percentage points increase. On the other hand, the total amount of consumer loans increased from 13% of GDP in 1967 to 0.16% of GDP in 1996, a 3 percentage points increase.

However, this figure could be misleading, because the figure shows the total amount of the *gross* debt of households and not the *net* debt of households. If households increase the gross asset holding and the gross debt holding simultaneously, it might be the case that the net debt, defined as the sum of the negative amount of the net wealth (asset minus debt) across households, did not increase over the period, even though the total amount of gross debt increased.

In order to address the issue, let us start by defining the notion of "non-housing wealth" and "non-housing debt" of households. Non-housing wealth is defined as the total wealth of households (total gross asset minus total gross debt) minus the value of homes. This represents the value of household's wealth other than the housing, and corresponds to the definition of the financial asset in the model, which will be discussed later. If a household holds a negative amount of the non-housing wealth, it is called the non-housing debt. Total amount of the non-housing debt is the sum of the non-housing debt across households.

Using the first (1983) and 1998 waves of the Survey of Consumer Finances (SCF), it is shown that there is an evidence that, at least between 1983 and 1998, the total amount of households debt measured by the total amount of non-housing debt relative to GDP increased as well. Table 1 summarizes the findings. The first row shows the same data as those used in the Figure 2, for comparison. The aggregate *gross* debt increased by 19 percentage points between 1983 and 1998. The second row indicates that, on average, households held \$5,263 (in 2000 U.S. dollars) of non-housing debt in 1983, but the amount increased to \$12,449 in 1998. The rate of the increase between the two years is 137% and is substantially higher than the rate of the increase of real GDP over the same period, which was 67%. The next row indicates that the total non-housing debt relative to nominal GDP, computed by multiplying the amount of debt per household by the number of households from Census data and dividing it by the nominal GDP of corresponding year, increased by 7 percentage points, from 9% to 16%. The level of the debt over GDP is lower than the data in the first row (and the data used for Figure 2), but it is obvious that both data show a significant increase between 1983 and 1998. An increase of the non-housing debt can be also inferred from the proportion of households in a negative non-housing wealth position. In 1983, the proportion was about 17%. However, in 1998, the proportion of households with a negative non-housing wealth increased to 28%.

Table 1: Aggregate Amount of Debt of U.S. Households

	1983	1998	Change
Total gross debt/GDP	0.442	0.629	+0.187
Total non-housing debt per household (2000 U.S. dollars)	5263	12449	+137%
Real GDP (billions of 2000 U.S. dollars)	5424	9067	+67%
Total non-housing debt/GDP	0.089	0.158	+0.069
Proportion of households with negative non-housing wealth	0.165	0.282	+0.117

[1] "Total non-housing debt" is defined by the sum of negative non-housing wealth of households.

[2] "Total non-housing debt/GDP" is constructed by multiplying the total non-housing debt by the number of households (from Census) and dividing it by the nominal GDP of each year.

3 Model

3.1 Overview of Model and Experiments

We use a dynamic overlapping-generations general equilibrium model with uninsured idiosyncratic earnings uncertainty. The distinct feature of the model is that there are two types of capital, namely housing (residential) capital and non-housing (non-residential) capital. The assets traded in the model are the shares of the two types of capital. Non-housing capital yields non-housing consumption goods, which can be either consumed or used to buy shares of capital. Housing capital yields a flow of housing services, which is an imperfect substitute of non-housing (standard) consumption goods. The next six sections describe the environment of the model.

Section 3.8 defines the steady state equilibrium of the model. We restrict our attention to the steady state equilibrium. We conduct a steady state comparison to measure the effects of the changes in the earnings process on the individual portfolio choice and the equilibrium asset prices.

3.2 Demographics

The demographics structure is assumed to be time-invariant. Every period, measure x_1 of agents is born. Each agent lives at most I periods. All agents of age i are subject to an exogenous survival probability $\psi_i \in [0, 1]$ from age i to age $i+1$. Since the survival probability is assumed to be identical across agents of the same age, measure of agents of age $i+1$ can be represented recursively as $x_{i+1} = \psi_i x_i$. x_1 is determined such that total measure of agents $\sum_{i=1}^{I} x_i$ is normalized to 1. Since there is no aggregate uncertainty in terms of the population turnover, the population size is 1 every period.

3.3 Technology

There are two kinds of capital, housing and non-housing capital. They are modeled as two trees, as in Lucas (1978). The two types of capital can be called the housing tree and the non-housing tree. The trees produce dividends. The dividends on the housing tree are housing services, and the dividends of the non-housing tree are non-housing consumption goods. These dividends are imperfect substitutes.

In order to make it easier to compare with the U.S. economy, the stock of housing and non-housing capital, or the size of the trees are assumed to be fixed at H, and K, respectively. Housing tree yields $d^h = R^h H$ units of housing services every period. Non-housing tree yields $d^k = R^k K$ units of non-housing consumption goods each period. The use of R_h and R_k makes it easier to relate the model to the U.S. economy when the model parameters are calibrated.

Since the fixed amount of housing and non-housing capital (fixed size of housing and non-housing tree) is a crucial assumption, it requires a discussion for justification. Glaeser, Gyourko and Saks (2004) show that the rise in the housing prices since the 1970s is due to the difficulty in obtaining the regulatory approval for building new houses. Davis and Heathcote (2004) decompose a housing price index into the price index for residential structures and the price index for the residential land. Using the decomposition, they obtain an index for the supply for the residential land. They found that *(i)* the growth rate of the real supply of the residential land has been very low (about 0.5% per year), and *(ii)* the price index for the residential land increased substantially since the 1970s.

For an agent of age i, holding of the shares is represented by (h_i, k_i), where $h_i \in \mathcal{H}$ and $k_i \in \mathcal{K}$. In order to relate the assets in the model to those in the U.S. economy more easily, We are going to call shares of the housing tree as housing assets, and shares of the non-housing tree as financial assets. Naturally, the price of the share of the housing tree, p^h, corresponds to the housing price, while p^k corresponds to the price of the financial (non-housing) asset.

Without loss of generality, the quantity of shares of housing and non-housing capital are normalized to be H and K, respectively. Because of the normalization, the unit price of each share, which is an equilibrium object, is p^h and p^k, respectively. Notice that the rate of return of the housing and non-housing asset in period t can be expressed simply as:

$$r^h = \frac{d^h}{H} = R^h \tag{1}$$

$$r^k = \frac{d^k}{K} = R^k \tag{2}$$

If an agent owns k_i financial asset, the agent receives $r^k k_i$ as dividends. If an agent owns h_i housing asset, the agent receives dividends $r^h h_i$ from the housing tree. There are five assumptions associated with the housing asset and their dividend, which enable the housing assets in the model to capture distinct characteristics of housing assets in the U.S. economy. The assumptions are as follows:

1. It is assumed that the dividends from the housing tree cannot be traded. The dividends correspond to the housing services that the owner of the housing can enjoy. Since we assume strictly positive marginal utility with respect to the housing services, it is always optimal for an owner of a housing asset to consume all the dividends (housing services) that are received. Formally, if we denote b_i as the consumption of housing services, enjoyed by the owner of housing asset h_i, the following must hold:

$$b_i = r^h h_i \tag{3}$$

2. Holding a negative amount of housing assets (*i.e.*, taking short position) is prohibited.

3. If an agent owns hi, the agent has to pay $p^h h_i \delta^h$ to maintain the quality of the housing capital.

4. Housing assets can be used as a collateral for a secured debt. In particular, if an agents holds h_i, the agent can borrow up to $p^h h_i(1-\lambda)$ where $\lambda \in [0, 1]$. Formally, the borrowing constraint associated with the secured debt is:

$$p^k k_i \geq -p^h h_i(1-\lambda) \tag{4}$$

   Notice that λ can be interpreted as the down payment requirement. If an agent wants to buy housing asset of size h_i, the agent has to show the proportion λ of the value of the house $p^h h_i$.

5. There is an mortgage interest rate premium associated with the secured debt backed by the housing asset.The premium is denoted as ι.

With these assumptions, shares of the housing capital capture the characteristics of owner-occupied housing. Shares of the housing capital can be owned but the dividends cannot be traded, so the owners optimally choose to consume the dividends that are received. Agents can only own positive amount of the shares of the housing capital, but can borrow in terms of financial asset, using the value of the shares of the housing capital as collateral. However, agents have to pay the down payment of at least $p^h h_i \lambda$ to hold the housing asset h_i. In addition, if an agent is borrowing against the value of housing assets, there is a mortgage interest premium of ι added to the saving interest rate. Finally, there is a maintenance cost associated with housing asset, which is a proportion δ^h of the value of the housing asset.

Shares of the non-housing capital corresponds to the risk-free financial asset, or mortgage debt, in case the amount is negative. There is no risk associated with the return of the asset, and there is a mortgage interest premium ι if an agent is taking negative position of the asset.

There is no unsecured debt in the model. This is effectively achieved by the assumption that if an agent files for a bankruptcy, the only thing that the creditors can confiscate is a proportion $1-\lambda$ of the housing asset which is used as collateral. If there is no other punishment and there is no commitment technology for agents, there would be no debt, except for the secured debt with housing as collateral.[2]

3.4 Endowment

Every period, each agent receives stochastic earnings. Earnings are represented by $y(i,z)$, where i represents an average life-cycle profile of earnings. $z \in \mathcal{Z}$ represents a stochastic component of earnings. z is assumed to follow a finite state first order Markov process $\Gamma_{zz'}$. The stochastic process of z will be specified such that it represents the earnings instability that is observed in the data. Details of the earnings process are presented in Section 4.2. Each agent is born with initial share holdings h_1 and k_1. They are exogenously chosen and set to zero.

3.5 Preference

An agent of age i derives utility from consumption of a non-housing goods c_i and housing services b_i. An agent values streams of consumption of both goods $\{c_i, b_i\}_{i=1}^{I}$ according to:

$$E_0 \left\{ \sum_{i=1}^{I} \beta^{i-1} u\left(c_i, b_i, i\right) \right\} \tag{5}$$

where β is a time discount factor and E_0 is an expectation operator with information available at the period 0 which is before the birth. The period utility function is assumed to be strictly increasing, strictly concave, and satisfy Inada conditions in the first two arguments. The third argument of the period utility function i represents the change in the household size over the life-cycle. More precisely, period utility for agents of age i is discounted by the reciprocal of ζ_i. $\{\zeta_i\}_{i=1}^{I}$ represents the average household size at age $i = 1, ..., I$ and is exogenously given. The period utility for being dead is normalized to zero.

3.6 Social Security System

There is a pay-as-you-go social security system. There is a mandatory retirement age $\hat{I}$. The government levies payroll taxes on workers (agents with $i \leq \hat{I}$) with a constant tax rate ω, and distributes the tax income to the retired agents (agents with $i > \hat{I}$). It is known that the social security benefit is positively but imperfectly correlated to the total earnings over the working

[2] There is a growing literature in macroeconomics on the unsecured debt, as the amount of unsecured debt and the number of people who default on their unsecured debt substantially increased since the beginning of the 1980s. For example, see Chatterjee, Corbae, Nakajima and Ríos-Rull (2002).

life.[3] In order to capture the features of the U.S. social security system without a prohibitive computational cost, it is assumed that the amount of social security benefits depends only on the value of the permanent shock to earnings to each agent. All the retired agents of the same realization of the permanent shock to earnings receive the same amount of benefits regardless of their age. To ensure the feasibility of the system, the benefit is determined so that the total amount of benefit is equal to the amount of social security tax income each period.

Without loss of generality, social security taxes and benefits are modeled as adjustments to the earnings for each type of agents. Earnings for the workers are reduced by a constant rate ω, which is calibrated to the payroll tax rate. Earnings for the retirees are increased from zero (the amount in the world without social security benefits) to the amount of the social security benefit, which depends on the permanent shock to earnings, and is determined such that the total amount of benefit is equal to the total tax amount every period.

3.7 Market Arrangement

Agents face two types of shocks, the shock to earnings, and the death shock. Earnings shock is assumed to be uninsurable, but complete annuity market exists for the death shock. If there is a mass of agents of the same age i, the survival probability ψ_i implies that, if agents of age i sign a contract to insure each other against the death shock, an agent of age i who saves $(\psi_i h_i, \psi_i k_i)$ and survives to age $i+1$ will receive (h_i, k_i), after sharing the wealth of the agents who were of age i but died before age $i+1$ among those who survived to age $i+1$.

3.8 Steady State Equilibrium

In this section, we define the steady state equilibrium, where the economic environment is time-invariant, and the equilibrium prices are invariant as well. Agent's problem is formalized recursively. All agents are price takers in all the markets. The individual state is characterized by age, realization of earnings shock, and holdings of the tree shares (i, z, h, k). Price of the non-housing consumption good is normalized to one. Time-invariant prices of shares p^h and p^k are relative to the price of the non-housing consumption good.

The recursive formulation of the agent's problem is as follows:

$$V(i, z, h, k) = \max_{c,h',k'} \left\{ u(c, b, i) + \beta\psi_i \sum_{z'} \Gamma_{zz'} V(i+1, z', h', k') \right\} \tag{6}$$

subject to

$$c + \psi_i p^h h' + \psi_i p^k k' = p^h(1-\delta^h)h + (p^k + r^k + \iota\mathcal{I}_{k<0})k + y(i, z) \tag{7}$$

$$h' \geq 0 \tag{8}$$

$$p^k k' \geq -p^h h'(1-\lambda) \tag{9}$$

[3]Conesa and Krueger (1999) analyzes the risk sharing effect of the U.S. social security system.

where $\mathcal{I}_{k<0}$ is an indicator function which takes value one if $k < 0$ and zero otherwise. $p^h \delta^h h$ denotes the maintenance cost of housing capital when the agent's housing asset holding is h.

Associated decision rules are $g_h(i,z,h,k)$, $g_k(i,z,h,k)$, and $g_c(i,z,h,k)$. The measure of agents of each type is defined as $m(i,z,h,k)$. The transition function that is consistent with optimal decision rules and law of motions of exogenous stochastic processes is defined as $T(m)$. Now the equilibrium is ready to be defined.

Definition 1 ***A steady state equilibrium*** *is a value function $V(i,z,h,k)$, associated decision rules $g^h(i,z,h,k)$, $g^k(i,z,h,k)$, $g^c(g,z,h,k)$, measure of agents $m(i,z,h,k)$, and prices (p^h and p^k) such that:*

(i) ***(Agent's optimization)*** *Given a set of prices $\{p^h, p^k\}$, $V(i,z,h,k)$ solves the functional equation described above and $g^c(i,z,h,k)$, $g^h(i,z,h,k)$, and $g^k(i,z,h,k)$ are the associated optimal policy functions.*

(ii) ***(Market clearing for housing assets)***

$$\int h \; dm \;=\; H \tag{10}$$

(iii) ***(Market clearing for financial assets)***

$$\int k \; dm \;=\; K \tag{11}$$

(iv) ***(Consistency and Stationarity)*** *For a transition function T which is consistent with the optimal decision rules of agents,*

$$m = T(m) \tag{12}$$

4 Calibration

4.1 Demographics

One period is a year, because the data set that is used to estimate the changes in the earnings process (Panel Study on Income Dynamics, PSID) has an annual frequency. The demographics is assumed to be time-invariant. Life of agents starts at age 20 (period 1), and agents can live up to age 99 (period $I = 80$), but there is an early death shock. The mandatory retirement age is set at age 59 ($\hat{I} = 59$). Conditional survival probability $\{\psi_i\}_{i=1}^{I}$ is taken from the survival probability of males in Bell, Wade and Goss (1992). Due to the assumption on the maximum length of life, $\psi_{80} = 0$. Figure 3 shows both the survival probability conditional on age and the unconditional survival probability.

In calibrating the household size $\{\zeta\}_{i=1}^{I}$, whose reciprocal is the discount factor for consumption, we take into account both the average age profile of the household size, and the intra-household economy of scale. First, the average household size conditional on the age of the head of households is computed from 1998 wave of the Survey of Consumer Finances (SCF). Second, the number is converted using the family equivalence scale computed by the U.S. Department of Health and Human Services. Specifically, I fit the mapping from the household size to the family equivalence scale into fourth order polynomial and use the polynomial function to convert the average household size of each age to family equivalence scale for each age.

Figure 4 shows the average household size conditional on age of the head of household, and converted into family equivalence scale, in order to take into account the intra-household economy of scale. The series show a hump shape over the life-cycle, which induces a hump in the life-cycle consumption profile (Fernández-Villaverde and Krueger (2001)).

4.2 Earnings Process

This section describes how to construct the earnings processes used in the model from the data of workers in the Panel Study on Income Dynamics (PSID). Section 4.2.1 describes how to estimate the time-varying earnings process, studies the properties of the earnings process, and compares the findings with other related literature. Section 4.2.3 discusses how to approximate the estimated earnings processes so that the processes can be used in the model.

4.2.1 Estimating Earnings Process

Earnings process is estimated from the PSID, using the method developed by Heathcote et al. (2003). I estimate the time-varying *earnings* process whereas they estimate the time-varying *wage* process. The difference reflects the difference of the focus: the current paper focuses on the changes in the individual's portfolio choice between housing and financial assets and the changes in asset prices, while theirs is the changes in the inequality of hours worked, earnings, and consumption.

As in Heathcote et al. (2003), only the white male head of households are included in the sample. The studies on the U.S. rising earnings inequality since the late 1960s mainly focus on the changed occurred to white males. As discussed in Heathcote et al. (2003), male earnings occupy on average 80% of the total household earnings, and the correlation between male earnings and the total household earnings is very high.

Using their estimation procedure, which is described to the details in the Appendix A, we basically estimate the following time-varying earnings process during the period of 1967 to 1996:

$$\log(y_{i,a,t}) = \bar{y}_{i,t} + \hat{y}_{i,a,t} \tag{13}$$

$$\hat{y}_{i,a,t} = \pi_t \gamma_a + \eta_{i,a,t} + \phi_t \nu_{a,t} \tag{14}$$

$$\eta_{i,a,t} = \rho \eta_{i-1,a,t-1} + \tau_t \epsilon_{a,t} \tag{15}$$

Equation (13) states that log earnings of an individual a of age i at period t can be disaggregated into two components. The first component $\bar{y}_{i,t}$ is the time-varying age-dependent average earnings profile. The second component $\hat{y}_{i,a,t}$ is the shock to earnings. $\bar{y}_{i,t}$ is fitted to the following fourth order polynomial in age.

$$\bar{y}_{i,t} = b_{0,t} + b_{1,t}i + b_{2,t}i^2 + b_{3,t}i^3 + b_{4,t}i^4 \tag{16}$$

Using the procedure, $\hat{y}_{i,a,t}$ is obtained as mean zero residuals out of the regression.

Equation (14) states that the shock to earnings $\hat{y}_{i,a,t}$ can be decomposed into three components. The first component is the *permanent* shock to earnings. This component is intended to capture what changes the level of earnings of an agent uniformly and permanently over the life-cycle. In particular, difference in earnings due to the difference in education, skills, or innate ability are expected to be captured by this component. γ_a is an *i.i.d.* random variable with zero mean and unit standard deviation. It is drawn only once at birth for each agent. Therefore, each agent has the same γ_a over the life-cycle. However, the loading factor for γ_a, which is π_t, is allowed to very over time. This is intended to capture, for example, the changes in the skill premium over time.

The second component of the shock to earnings, $\eta_{i,a,t}$, is the *persistent* shock to earnings. $\eta_{i,a,t}$ is assumed to follow AR(1) process with the persistence parameter ρ. Equation (15) formally specifies the process. The inclusion of the persistent component is motivated by the fact that the autocorrelation function of earnings declines with the length of lag, after the first lag (Gottschalk and Moffitt (1998)). ρ is assumed to be time-invariant. $\epsilon_{a,t}$ is an *iid* innovation term with mean zero and unit variance. The time-varying loading factor τ_t controls the size of the variance of the innovation term.

The last component of the shocks to earnings, $\phi_t \nu_{a,t}$, is the *transitory* shock to earnings. This component is motivated by the drop in the autocorrelation function of earnings at the first lag (Gottschalk and Moffitt (1998)). $\nu_{a,t}$ is a *i.i.d.* random variable with mean zero and standard deviation being one. ϕ_t is the time-varying loading factor, which controls the size of the transitory shocks to earnings in each period.

Following Heathcote et al. (2003), the time-varying parameters of $\hat{y}_{i,a,t}$ are estimated using the method proposed by Chamberlain (1984). Basically, the parameters are chosen such that weighted sum of the distance between each element of the covariance matrix implied by the model and the corresponding element of the covariance matrix obtained from the data is minimized. The estimation procedure gives estimates for $\{\bar{y}_{i,t}\}$, $\{\pi_t\}$, ρ, $\{\tau_t\}$, and $\{\phi_t\}$, for the period between 1967 to 1996.

4.2.2 Properties of the Estimated Earnings Process

Figure 5 exhibits the change in the cross-sectional variance of log earnings during 1967-1996 of the PSID sample. As found in a large number of literature, including Gottschalk and Moffitt (1994), Gottschalk and Moffitt (1998), Haider (2001), Katz and Autor (1999), and Heathcote et al. (2003), a substantial increase of *inequality* of earnings is observed during the period.

Figure 6 shows the time series of the variance of residual of log earnings after the average life-cycle component is removed. The average difference between the two is around 12%. It means that, the average earnings profile can explain on average 12% of the inequality of earnings, and that the remaining 88% of the earnings inequality still remains after taking into account the inequality due to the life-cycle earnings profile. More importantly, the upward trend of the time series of the earning inequality is irrelevant to the inequality due to the life-cycle earnings profile. The trend is evident in both the time series of the simple log variances or the time series of the variance of residual of log earnings.

Figure 7 shows the average life-cycle profile of earnings estimated from PSID, pooled across different years. The figure confirms the existence of a hump in the life-cycle earnings profile. The average earnings increases with age, peaks at around age 45, and then gradually declines until the retirement.

Figure 8 exhibits the changes in the standard deviation of the shocks over the period 1967-1996, estimated from PSID. For the case of the persistent component, the standard deviation of the innovation term is shown. The estimate for the persistence parameter is 0.9477. It is extremely persistent. It is consistent with estimates of the processes which do not have a random walk component. For example, Heathcote et al. (2003) obtained 0.9426, albeit for wage process. For other studies which use a earnings process with a random walk component, the estimate tend to be lower, because part of the variances attributed to the persistence shock in the model without a random walk process is now attributed to the random walk process. For example, the estimate of Gottschalk and Moffitt (1994) is 0.720.

Figure 9 exhibits the changes of the variances of each of the three components between 1967 and 1996. The changes in each of the three components of earnings shocks during 1967 to 1996 can be summarized as follows:

1. The variance of the permanent shocks to earnings showed a rapid increase during 1967 to 1974. It has been still on an upward trend during 1974 to 1996, but the speed of the increase slowed down.

2. The cross-sectional variance of the persistent shocks to earnings declined during 1967 to 1974, because the size of the variance of the innovation term is lower between 1967 to 1974 compared the size of the variance of the innovation term in 1967.[4] Between 1974 and 1996, the cross-sectional variance of the persistent shocks to earnings increased substantially. This change corresponds to the increase of the size of the variance of the innovation term since 1974, after stagnating between 1967 and 1974.

3. The variance of the transitory shocks showed a relatively large increase between 1967 and 1973. It was on an upward trend until mid-1980s, but experienced ups and downs during 1985 to 1996.

[4] In estimating the variance of the innovation term of the persistent process, constant variance of the innovation term before 1967 is assumed, as in Heathcote et al. (2003).

The literature studying the rise in the earnings or wage inequality since the late 1960s has been focusing on decomposing the rise in the earnings or wage inequality into the rise in instability and the rise in permanent inequality. This is because the two can have very different macroeconomic and welfare implications. If a rise in the earnings inequality is mainly due to a rise in the permanent inequality of earnings, the inequality in earnings implies the consumption inequality as well. On the other hand, if a rise in earnings inequality is mainly induced by an increase in earnings instability, the magnitude of the change in consumption inequality depends on the degree of completeness of market and how each individual can insure against the earnings instability (Attanasio et al. (2002)).

Haider (2001) uses PSID data between 1967 to 1991, and decomposes the changes in earnings inequality into the persistent inequality and the earnings instability. It is shown that *(i)* the earnings instability increased between 1976 and 1983, dropped and stayed low during the rest of the 1980s, and picked up in 1991, and *(ii)* persistent inequality increased significantly since the end of 1970s. Since the specification of the earnings process is different from ours, it is difficult to compare his results with ours directly. But his persistent inequality shows a similar pattern as our persistent component; both dropped until the early 1970s and kept increasing since then. The increase in his persistent inequality slowed down in the late 1980s, while our persistent element does not slow down. But if we assume that his persistent inequality contains both of permanent and persistent components in our specification, they might be consistent, because the variance of the permanent component drops in the late 1980s in our specification.

Gottschalk and Moffitt (2002) also use PSID data between 1967 to 1996, to decompose the changes in the earnings inequality. They use the persistent component which follows a random walk, and the transitory component which follows an ARMA(1,1) process. They find that *(i)* the variance of the persistent component dropped in the mid 1970s, increased dramatically in the early 1980s, and dropped again in the mid 1990s, and *(ii)* the variance of the transitory component remained stable during the 1970s, increased during the early 1980s, dropped temporality in the early 1990s. Again it is a difficult task to compare our results with theirs, because of the difference in the specification. But our results and theirs are consistent in that the increase in the earnings inequality has more persistent nature, if we assume that their persistent component contains both of our permanent component and most of our persistent component.

Since our estimation strategy is very close to that of Heathcote et al. (2003), the results obtained here is very close to theirs, even though they estimate *wage* process instead of *earnings* process. They find that *(i)* the variance of the permanent component increased during 1967 to the mid 1980s and slowed down, *(ii)* the variance of the persistent component declined until the mid 1970s and increased since then, *(iii)* the variance of the transitory component increased between 1967 and mid 1970s, remained relatively stable until the early 1990s, increased sharply in the early 1990s. The difference of focus between earnings and wage has two implications. First, because of the positive correlation between wage and hours worked, the *level* of earnings inequality is larger than that of the wage inequality. Secondly, due to the increase of the correlation during 1967 to 1996, the magnitude of the *increase* of earnings inequality is larger than that of the wage inequality.

4.2.3 Constructing Earnings Process for the Model

Since the focus of the paper is to study the implications of the changes in the earnings process on the portfolio choice of agents and equilibrium asset prices, the average life-cycle earnings profile is fixed throughout the experiments. Specifically, I estimate the *time-invariant* life-cycle average earnings profile $\tilde{y}_i$ by fitting PSID data which are pooled across the periods 1967-1996 to fourth order polynomial in age.

For the analysis of steady state equilibria, we want to compare a steady state equilibrium where the earnings shock is calibrated to the U.S. economy in 1967, with another steady state equilibrium where the earnings shock is calibrated to the U.S. economy in the 1996. Comparing the macroeconomic aggregates of the two economies enables us to measure the effect of the changes in the earnings instability on macroeconomic aggregates. On the other hand, all the parameters associated with the earnings shock except for the persistence parameter ρ are estimated for each of the sample years 1967-1996. In order to use the parameter estimates for the steady state comparison, we first calibrate the model economy to the 1967 U.S. economy.

The parameters of the earnings process that are used for the steady state comparison is summarized in Table 2. We use the parameters of the earnings process estimated for 1967 from PSID. This choice is justified by *(i)* the empirical findings using other data that the earnings process was stable prior to the late 1960s, and *(ii)* stable earnings process is assumed in the estimation procedure. As for the 1996 earnings process, we use the average of the last five years (1992-1996) for the parameters associated with the permanent and transitory components, and choose the standard deviation of the innovation term of the persistent component such that the cross-sectional variances of the residuals of log earnings of the model is equal to the average of the last five years of the cross-sectional variances of the residuals of log earnings of PSID data. Figure 10 compares the cross sectional variances of the residuals of log earnings of PSID data and those for 1967 and 1996 model economies. The figure guarantees that the cross-sectional variances of earnings used for the model economy in 1996 is a good representative of the average cross-sectional earnings variances between the late 1980s and the mid 1990s.

Table 2: Estimated Parameter Values for Earnings Process

Year	π	ρ	τ	ϕ
1967	0.2115	0.9477	0.1483	0.2007
1996	0.3403	0.9477	0.1669	0.3130

[1] Source: Estimated from PSID.

All of the three components of the shocks are continuous variables, while the earnings shocks are modeled as discrete in the model. In order to fill the gap, the shocks that are estimated are discretized.

The permanent shock to earnings is approximated using two grid points. In particular, the shock takes one of $(-\pi, \pi)$ with the equal probability. Similarly, the transitory component is approximated using two grid points. The shock can take one of $(-\phi, \phi)$ with the equal probability.[5]

The persistent shock to earnings is discretized using the procedure developed by Tauchen (1986). Five grid points are used as a compromise between the computational feasibility and the quality of approximation.[6] Grid points are symmetrically located with equal distance. We can denote the distance between grids as $\xi\tau$, where τ is the standard deviation of the innovation term of the AR(1) process, and ξ is the scale factor. Using the notation, the grid points can be expressed as $(-2\xi\tau, -\xi\tau, 0, \xi\tau, 2\xi\tau)$, and we have a free parameter ξ. ξ is pinned down such that, if all the agents start from $\eta = 0$, the cross-sectional standard deviation of η after 40 periods (time of mandatory retirement in the model) of the discretized process is close to that of the original AR(1) process. The procedure gives $\xi = 1.680$. Figure 11 compares the cross-sectional standard deviation of original AR(1) process with that of the approximated process with 5 grid points.

If we combine all of the three types of shocks to earnings, the earnings process can be expressed as 20-state first order Markov process for the agents before retirement age, and 2-state first order Markov process for agents after the retirement age. There are two states for the retired because the permanent shock to earnings is assumed to affect the social security benefit, but all the other shocks no longer matter after the retirement.

Finally, one adjustment to the average life-cycle earnings profile has to be made. This is because the shock process is estimated using the residuals of the log-earnings and thus the expected value of the shocks to earnings is not one. Therefore, the average life-cycle profile $\tilde{y}_i$ is adjusted such that the unconditional expectation of the life-cycle earnings profile of the model coincides with $\tilde{y}_i$, which is directly estimated from PSID.

4.3 Technology

The parameters related to technology are calibrated to mimic the U.S. economy in 1967, which is the first year of the PSID sample, with one important modification; we try to replicate the U.S. economy excluding top 1% richest with respect to the total wealth holding. It is well known that PSID under-represents the top end of the wealth distribution (Juster, Smith and Stafford (1999)). If we stick to our earnings process which is tightly estimated from the PSID sample, we cannot replicate the top end of the wealth distribution, unless we rely on an unrealistically low discount factor, or other unrealistically strong saving motive.[7] Moreover, if we keep the estimated earnings process but attains the level of savings, changes in savings and portfolio choice decision

[5]It is found that increasing the number of grid points for the shocks, while keeping the cross-sectional variance for each of the shocks, does not change the quantitative results significantly.

[6]It is found that increasing the number of grid points, while keeping the cross-sectional variance, does not change the quantitative results significantly.

[7]Castañeda, Díaz-Giménez and Ríos-Rull (2003) shows that it is possible to replicate the wealth inequality of the U.S. economy using a model with uninsured idiosyncratic earnings shock, but the earnings process is much more skewed than then one estimated from PSID. In addition, their model does not have a life-cycle features, so it is difficult to relate their result to our calibration.

by agents in response to the change in earnings instability, which is the object of the main focus of the paper, is expected to be distorted.

Table 3 shows the aggregate statistics of the U.S. economy in 1967 after the contribution from the top 1% richest is removed. These values are used to pin down the technology-related parameters. Details of the procedure to remove the richest 1% of agents are described in Appendix B.

Table 3: U.S. economy in 1967 excluding the richest 1% and Calibrated Parameters

Statistics	Value
Output (normalization)	1.0000
Non-Housing capital stock	1.5080
Housing capital stock	1.0631
Total capital stock	2.5711
Capital share	0.1767
Labor share	0.8233
Depreciation rate of non-housing	0.0708
Depreciation rate of housing	0.0161
Non-housing net return	0.0464
Housing net return	0.0789

[1] Source: Constructed from various data of BEA.

The procedure of how to calibrate the steady state equilibrium to the U.S. economy in 1967 is described below. We start by defining the aggregate earnings E in the steady state equilibrium as follows:

$$E = \int y(i,z)\, dm \tag{17}$$

The computed aggregate earnings and the labor share enables us to compute the size of the total output Y under the labor share in the data. Y can be computed by $Y = \frac{E}{0.8233}$. Using Y, the steady state stock of housing and non-housing capital stock, H and K, can be calibrated as $H = 1.0631Y$ and $K = 1.5080Y$. r^h and r^k are directly obtained from the return of the housing and non-housing capital. Specifically, $r^h = 0.0789$ and $r^k = 0.0464$. The dividends d^h and d^k are obtained by applying the definition of the dividends, using the capital stock and the rate of return. Finally, the maintenance cost parameter of the housing asset is calibrated using the depreciation rate, that is, $\delta^h = 0.0161$ The down payment requirement λ is set to 20%, which is the average down payment ratio of the primary mortgage loans in the U.S. Mortgage premium ι is set to 1.59%, which is the average difference between the average interest rate of 30 year conventional mortgage, and the average interest rate of the 30-year treasury bill. The difference had been stable over time.

4.4 Preference

The period utility function is assumed to be of Constant Relative Risk Aversion (CRRA) type. Besides, the Cobb-Douglas aggregator is used to aggregate the consumption of non-housing consumption goods and housing services. In sum, the utility function is given by:

$$u(c,b,i) = \frac{(\frac{1}{\zeta_i}\ c^{\theta} b^{1-\theta})^{1-\sigma}}{1-\sigma} \tag{18}$$

As the benchmark case, a coefficient of risk aversion σ is set to 2. This is the median value in the literature. Cobb-Douglas aggregator function can be considered as the special case of the functions of constant elasticity of substitution (CES). Fernández-Villaverde and Krueger (2001) use a CES function as an aggregator of durable and non-durable consumption, and argue that assuming elasticity parameter to be 0 (so that the CES function becomes the Cobb-Douglas function) is a reasonable choice, according to literature that estimates this parameter.

How are the discount factor β and the aggregation parameter θ calibrated? Instead of using direct evidences to calibrate the two parameters, we employ the exactly-identified Generalized Method of Moments (GMM); we choose the values of the two parameters to exactly match the following two targets:

1. The total value of housing asset over Y is equal to the corresponding value of the 1967 U.S. economy. Thanks to the way we specify the technology, it is satisfied if $p^h = 1$.
2. The total value of non-housing asset over Y is equal to the corresponding value of the 1967 U.S. economy. Thanks to the way we specify the technology, it is satisfied if $p^k = 1$.

Because of the non-linear nature of the problem, there is no guarantee that the parameter values that satisfy the two conditions simultaneously exist, but we successfully found the values of β and θ that achieves the targets; The calibration procedure gives $\beta = 0.9479$ and $\theta = 0.9219$

4.5 Social Security System

Following Conesa and Krueger (1999), the constant payroll tax rate is set at 15.3%. The implied replacement rate in the 1967 economy is 55%, which is within the reasonably close range of the average replacement ratio of the U.S. economy.

5 Computation

The problem of an agent is solved using value function iteration, given a set of price (p^h, p^k). The value function has four arguments (i, z, h, k). In addition to the two discrete individual states (age i and the shock to earnings z), there are two continuous individual state variables, which characterize the asset holding of an agent (h, k). I approximate the space of housing asset using discretization, and restrict the choice to the set of the discrete grid points. We implement the grid search in searching the optimal h'.

As for the financial asset space, I approximate the value function, conditional on (i, z, h), using shape-preserving spline approximation, developed by Schumaker (1983).[8] There are two substantial benefits of the approximation method, which improves computational stability; first of all, the approximation method preserves the monotonicity and the concavity of the original function. Combined with the strict concavity of the utility function, the search of the optimal level of k' is very stable and the unique optimal k' is always guaranteed. Secondly, once the approximate function is obtained, there is almost no cost in evaluating the derivative of the approximate function because the approximate function is a collection of second order polynomials.

Partly because the size of individual state space is very large (there are 80 ages, and, for working agents, there are 40 potential earning levels), and partly because there are two continuous choice variables (housing and financial asset holding), the computational burden of solving individual agent's problem given a set of prices is extremely large. In addition, thee is another loop to find an equilibrium set of prices. We can solve the problem by using the parallel computation. Specifically, Fortran 90 with MPI (Message Passing Interface) Library running on a Beowulf cluster is used. MPI is an extension of fortran 90 and allows to parallelize a fortran program and thus makes the execution significantly faster than single processor environment. Appendix C contains more details about the computational procedure.

6 Results: Properties of the 1967 Model Economy

6.1 Life-cycle Profiles

Figure 12 shows the average life-cycle profile of earnings and consumption. The average life-cycle profile of earnings with adjustments associated with the social security contribution and benefit is also shown. The life-cycle profile of earnings exhibits a hump shape, as implied by Figure 7. After the mandatory retirement age of 60, agents receive on average about 40% of the earnings after social security contribution at retirement. Clearly, the average life-cycle consumption profile is smoother than that of earnings. Agents on average increase consumption in their first 30 years, slowly reduce the consumption until age 80, and have a relatively flat consumption profile after age 80.

Figure 13 shows the average life-cycle profile of total wealth holding. It has a peak at the retirement age, implying a strong life-cycle saving motive. Figure 14 shows the decomposition of the total wealth into financial and housing assets. Clearly, the difference of the profiles is significant. Until around age 35, on average, agents take a negative position in terms of the financial asset, and accumulate the housing asset. Agents on average accumulate sufficient

[8] Judd (1998) summarizes the method.

amount of housing asset by around age 55. It is consistent with the young households behavior in the U.S. economy of trading up of the housings, and borrowing using secured mortgage loans. After age 35, agents on average finish repaying the debt and start saving in the financial asset, to prepare for the life after the retirement. After the retirement, agents dissave in terms of the financial asset to sustain the non-housing consumption, but do not reduce significantly the level of the housing asset holding. Agents reduce housing asset holding slowly as the average household size shrinks. This is consistent with the consumption profile in Figure 12, and also consistent with the findings of Venti and Wise (2004) that the majority of the retired households in the U.S. do not trade down their housing, unless they are hit by a significant shock like losing a household member.

6.2 Earnings and Consumption Inequality

Figure 15 shows how the cross sectional variances of log of earnings and consumption change over the life-cycle. Most importantly, the consumption inequality is significantly lower than the earnings inequality. This implies that a large fraction of the earnings inequality are due to the earnings instability and thus can be insured away.

The variance of log earnings increases until around age 40, because of the cross sectional variance of the persistent component starts from zero and almost converges to its stationary distribution. After the age 40, the variance of the log earnings remains stable until the retirement age.

The variance of log consumption increases gradually until the retirement age, in tandem with the increase in the cross-sectional variance of the persistent shock to earnings. After the retirement age, the consumption inequality drops slightly. Increasing consumption inequality among workers over the life-cycle is a property of the U.S. economy, as documented by Storesletten, Telmer and Yaron (2004).

6.3 Comparison with the Model without Housing

In order to highlight the properties of the baseline model economy with housing and financial assets, we construct another economy which is calibrated in a similar manner, but there is a key difference; there is no housing asset in the economy. We use the same earnings profile for 1967, but we use the value of the total capital stock , which is the sum of residential and nonresidential capital stock, in 1967 as the target to be matched by the total value of the financial asset in the economy. We recalibrate the discount factor (β) so that the value of total wealth to the total value of output in 1967 is the same as in the U.S. economy in 1967. The value of β calibrated for this economy is 0.9587.

Figure 16 compares the life-cycle profile of the total wealth of the economies with and without the housing asset. The young agents in the economy with housing save relatively more than in the economy without housing, and the middle-aged and the old agents save relatively more in the economy without housing. Agents save more in the economy with housing in order to accumulate as much housing asset as to support desired consumption of housing services as soon as possible. On the other hand, larger proportion of the consumption of the middle-aged and retired agents come from the consumption of housing services, and these agents don't need to save as much as in the economy without housing to support non-housing consumption in their life after retirement.

7 Results: Steady State Comparison between 1967 and 1996 Model Economies

In this section, changes in the portfolio allocation of individual agents, steady state equilibrium asset prices, and other macroeconomic aggregates are examined. The baseline experiment is to replace only the earnings process from the one estimated for 1967 U.S. economy to the one for the 1996 U.S. economy, and to compare the properties of the steady states of the model economy. In addition, the changes in response to a change in each of the three components of earnings process are also examined, to relate the various changes in macroeconomic aggregates to different components of earnings process.

7.1 Portfolio Choice

Figure 17 compares the average profile of consumption and earnings in the 1967 and 1996 model economies. Clearly, there is no significant differences between the two.

Figure 18 compares the average portfolio choice (denominated in terms of non-housing consumption goods) for each age in the two steady states, one with the 1967 earnings process and the other with the 1996 earnings process. As for the financial asset, working agents in the 1996 economy on average increase the financial asset holding more than in the 1967 economy. The 1996 earnings process is characterized by larger volatility of each of the three components of earnings process, and thus the working agents are those who have incentives to increase savings for precautionary purpose. On the other hand, savings for the retired are for life-cycle purpose. Since the average earnings profile is same in the two model economies, as shown in Figure 17, there is little incentive for the retired agents to change their financial asset holding profile.

Regarding the housing asset holding, the first thing to notice is that the way the life-cycle housing asset holding profile changes is different from the changes in the financial asset holding. This is primarily because of the dual nature of housing assets. On the one hand, the housing asset is a means of saving, so it is natural that the housing asset holding increases when there is a higher incentive for saving in general. However, on the other hand, the housing asset is also a source of housing services consumption. As long as the consumption profile of non-housing goods does not change significantly, as seen in Figure 17, the agents want to keep the housing asset holding unchanged as well. The second element, which separates the housing asset from financial assets, create the difference in changes between the housing asset holding and the financial asset holding.

Figure 18 also implies that changes in housing asset holding are not concentrated in any particular age group. The changes are more widespread across different age groups. This is because the changes in the housing asset holding is mainly induced by the general equilibrium effect. A higher volatility in individual earnings does not induce higher savings in the form of housing asset, but when the rate of return of the financial asset drops due to the higher demand for the asset, the return of the housing asset becomes relatively attractive to agents, and there is a shift of asset portfolio from the financial asset to the housing asset. Since the relatively higher return of the housing asset attracts agents regardless of their age, the changes in the housing asset holding is not concentrated in any particular age group.

7.2 Asset Prices and Other Macroeconomic Aggregates

Table 4: Comparison of macroeconomic aggregates: 1967 and 1996 model economies

Statistics	1967 model economy	1996 model economy	Change
Output (normalized)	1.000	1.000	–
Total housing asset	1.063	1.155	+8.7%
Total financial asset	1.508	1.619	+7.4%
Total wealth	2.571	2.774	+7.9%
Total debt	0.122	0.106	-13.3%
Price of housing asset	1.000	1.087	+8.7%
Price of financial asset	1.000	1.074	+7.4%
Interest rate	4.64%	4.32%	-0.32%

[1] Size of the change in the interest rate is in the percentage point.

Table 4 summarizes the changes in macroeconomic aggregates between the 1967 and 1996 model economy. Most strikingly, the changes in the prices of assets are close to 10%, even though the only thing happened between the two economies is a change in the individual earnings volatility. Specifically, the housing price increases by 9% in response to the change in the earnings process, and the price of the financial asset increases by 7%. Since the quantity of housing and non-housing capital are fixed, the changes in prices of assets (shares) directly affect the value of total assets in the economy. The total value of housing and financial asset held by the agents increase by 9% and 7%, respectively. Total wealth held by the agents increases by a little bit more than 8%, which is a weighted average of the changes of the two kinds of assets.

The change in the housing price and the change in the price of the financial asset is different because the housing asset is not a perfect substitute for the financial asset. Housing asset is held not only for the saving purpose but also for the consumption purpose. Since the total amount of non-housing consumption is fixed, agents do not want to change their housing asset holding, even though, as a means of saving, agents want to increase the housing asset holding. Indeed, the aggregate demand for the housing asset increases much less than the increase in the demand for financial assets, in response to the rising earnings volatility, when the interest rate associated with the financial asset is unchanged. Table 5 summarizes the changes in asset portfolio in economies with and without the general equilibrium effect. Figure 19 exhibits the average changes in the demand for assets when the 1996 earnings process is employed but there is no general equilibrium effect. As Figure 19 confirms, if the rate of return of the financial asset is unchanged, agents want to satisfy the additional saving motive due to an higher volatility of earnings mostly by increasing the financial asset holding. However, the general equilibrium effect pushes down the rate of return of the financial asset, in response to the increase in the demand. Finally agents shift their portfolio to the housing asset.

Table 5: Change in demand for assets

Statistics	1967 model economy	1996 model economy	
		without GE effect	with GE effect
Output (normalized)	1.000	1.037	1.000
Total housing asset	1.063	1.100	1.155
Total financial asset	1.508	1.814	1.619
Total wealth	2.571	2.914	2.774
Interest rate	4.64%	4.64%	4.32%

[1] All the statistics are normalized by the output in equilibrium, which is the same both for the 1967 model economy and for the 1996 model economy with the general equilibrium effect.

7.3 Changes in the Housing Price

The model prediction about the housing price is consistent with the trend in the U.S. housing price data qualitatively, and quantitatively. The baseline experiment shows a 9% increase in the equilibrium housing price. The number is close to 40$ of the change in the housing price index between 1967 to 1996. This number is big, considering that the only source of the change is the rise in the volatility of individual earnings, and there are other factors which push up the housing prices, like the aging of the baby boomers.

The predicted rise in the housing price in the baseline experiment is still substantially smaller compared with the recent rise in the housing price. The housing price index increased by 78% between 1967 and 2004, whereas the model prediction is 9%. However, there is a potential channel that the model can produce even larger increase in the housing price. It is related to the sensitivity of housing price to the interest rate. Remember that the most of the increase in the housing demand in the model experiment is induced by the drop in the interest rate due to the higher demand for the financial asset. However, the actual drop in the interest rate in the model is mare 0.3 percentage point. In other words, the demand of housing asset is very sensitive to the changes in the return of financial asset. On the other hand, the timing of the rapid rise in the housing price is concurrent with the drop in the stock price, which could be induced by the expected rate of return of nonresidential capital. If there is a drop in the rate of return of nonresidential capital, due to a technology shock, in the late 1990s, the model can predict a huge rise in the housing price, due to the shift of demand between the financial asset to the housing asset.

7.4 Changes in the Total Amount of Debt

On the empirical side, the total amount of gross debt increased dramatically since mid 1980s, as shown in Figure 2. On the theory side, in an economy with complete markets, a mean preserving increase of intra-group volatility of earnings implies a larger total gross debt. A natural guess is that an increase in the volatility of earnings might increase the total amount of debt in an incomplete market economy, too.

However, the result from our baseline experiment showed the opposite to such intuition. The fifth row in Table 4 shows that the total amount of debt actually *decreases* with an increase in the earnings volatility. This is because the decline of the debt due to an increase in the savings for precautionary motives dominates the increase of debt due to the higher borrowing demand by those who receive temporary lower earnings. Figure 20 confirms that the young agents in 1996 model economy try to save more than in the 1967 model economy.

Although it is impossible to formally discuss the timing of events using our experiments, the timing when the debt started to increase does not seem consistent with the timing of increases of volatility of earnings. The debt was approximately flat until the mid 1980s and started to pick up quite rapidly, whereas the increase in the earnings volatility is not concentrated in the period after the mid 1980s. Rather, the timing implies that the rise in the amount of debt might be related to the development in the financial sector, many of which occurred during the early 1980s.

7.5 Consumption Inequality

Table 6 shows how the inequality across agents changes between the 1967 and 1996 model economy, measured by the cross-sectional standard deviation of log. The numbers represent the inequality only among the workers, in order to be consistent with the data that are compared to the results. In addition, inequality arising due to the difference in age or household size is removed for the same reason. Expenditure on the housing services is imputed by multiplying the housing asset holding by the sum of the rate of return of the financial asset and the depreciation rate of housing capital. There are two parts in the table. The top half shows the results from the baseline experiment, using the model with housing and financial assets. The bottom half of the table shows the results from the model without housing asset.

Table 6: Changes in inequality in the model economies

	1967 model economy	1996 model economy	Change
Economy with housing			
Earnings	0.2639	0.4416	+0.1777
Non-housing consumption	0.1811	0.2928	+0.1117
Housing asset	0.2763	0.4163	+0.1400
Total wealth	1.0843	1.3094	+0.2251
Imputed total cons exp	0.1831	0.2950	+0.1119
Economy without housing			
Earnings	0.2639	0.4416	+0.1777
Non-housing consumption	0.1786	0.2868	+0.1082
Total wealth	1.8625	1.7738	-0.0887

[1] All are represented in the standard deviation of log, among workers, after removing the differences due to age or household size.

Table 7: Changes in inequality in the U.S. economy

	1972-73	1980-81	1985-86	1990-91	1997-98
Nondurable cons expenditure	0.0	-4.6	-2.6	-4.5	-4.0
Nondurable + durable cons exp	0.0	NA	1.7	0.6	1.5
Total expenditure	0.0	0.4	7.9	5.6	7.4

[1] Replicated from Krueger and Perri (2003a).
[2] Their source is Consumption Expenditure Survey.
[3] Unit is the percentage point changes in the standard deviation of consumption expenditure.

Table 7 is taken from Krueger and Perri (2003a). The table shows the changes in percentage point of the cross-sectional standard deviation of log of three definitions of consumption between 1972-73 and 1997-78. They use Consumption Expenditure Survey (CEX) data. They use households whose reference person is of age 25 to 64 with positive earnings and consumption, measure the changes in the part of inequality that cannot be attributed to the differences in either age, race, or household composition. The first row is associated with nondurable consumption expenditure only. The second row takes into account both the nondurable consumption expenditure and the imputed services consumption of housing and car. The third row is associated with the total consumption expenditure.

As expected, the inequality of earnings increased, from 0.26 in the 1967 model economy to 0.44 in the 1996 model economy. Non-housing consumption equality is lower than the earnings inequality in each year, implying the consumption smoothing behavior of agents, and the change in the consumption inequality is smaller than that of the earnings. While the earnings inequality increased by 18 percentage points, non-housing consumption inequality increased from 0.18 to 0.29, by about 11 percentage points.

Although the output of the experiment is qualitatively consistent with the empirical findings of Krueger and Perri (2003a), the model produces a rise in the consumption inequality higher than the data reported by Krueger and Perri (2003a), and the number produced by experiments in Heathcote et al. (2003). As seen in Table 7, they reported an increase of 1.5 percentage points in the total consumption between 1972-73 to 1997-98, using CEX data. The baseline experiment in Heathcote et al. (2003) produces an increase of around 5 percentage points.

There are two reasons why the change in the consumption inequality produced by the model is higher than the numbers reported in Krueger and Perri (2003a). Firstly, as argued in Heathcote et al. (2003), we use change in the *earnings* inequality instead of *wage* inequality. If there is also an increase in the correlation between the wage and hours worked, the estimated change in the earnings inequality is higher than the estimate of the change in the wage inequality. Secondly, we are comparing two steady states. This implies that the earnings process in 1996 lasts long enough so that the economy reaches the steady state associated with the 1996 earnings process.

If we compare the top and the bottom half of Table 6, it is found that the change in the consumption inequality using the model without housing asset is close to the number produced by our baseline experiment. The standard deviation of log consumption increased by almost 11 percentage points in the experiment with the model without housing asset, too. This result implies that the change in consumption equality in response to a change in the earnings inequality is not substantially affected by the existence of housing asset in the model.

Although the existence of the housing asset does not change the effects of rising earnings inequality on the consumption inequality in our experiments, the data reported by Krueger and Perri (2003a) imply the opposite. Table 7 shows that, while the non-durable inequality *declined* by 4 percentage points, between 1972-73 and 1997-98, the inequality of nondurable plus imputed durable consumption increased by 1.5 percentage points, and the inequality of the total expenditure increased by 7.4% during the same period.

This might be due to the difference between the housing and the durable goods; car. While car is not included in the definition of the housing asset, car is an important part of durable goods. Though the average value of cars is smaller than the average value of housing assets, cars depreciate faster, implying that the value of services derived from cars is higher conditional on the value. Considering that the inequality of housing asset increased more than the non-housing consumption inequality, it is expected that the inequality of the total consumption (including both nondurable and durable goods) increase more than the inequality of the nondurable goods, if the contribution of the durable goods, including that from cars, is higher than in the current experiments.

Table 8: Decomposition of changes in inequality in the model economies

	1967 model economy	1996 model economy	Change
Economy with housing			
Earnings	0.2639	0.4416	+0.1777
Non-housing consumption	0.1811	0.2928	+0.1117
Housing asset	0.2763	0.4163	+0.1400
Total wealth	1.0843	1.3094	+0.2251
Imputed total cons exp	0.1831	0.2950	+0.1119

[1] All are represented in the standard deviation of log, among workers, after removing the differences due to age or household size.

8 Concluding Remarks

In the paper, we investigate the effects of well-documented rising earnings inequality on the portfolio choice and asset prices. In particular, we measure the effects of rising earnings inequality on *(i)* the housing price, *(ii)* the size of the mortgage debt, and *(iii)* the consumption inequality. As for the first question, it is found that the rise in the earnings inequality can produce a substantial fraction of the rise in the housing price observed in the U.S. data. As for the size of secured debt, the experiments imply that the rise in the earnings inequality *reduces* the size of the mortgage debt, contrary to the data, because of the increased demand for the precautionary savings. As for the last question, we find that models with and without the housing market give a very similar answer to the effect of the rising earnings inequality on the consumption inequality, implying that the role of housing asset on the consumption inequality is limited.

A promising extension, which is under way in Nakajima (2005), is to explicitly model the development of the U.S. financial sector over the last 30 years. Our experiment results imply that this might be the primary cause of the rapid increase in the mortgage debt. Moreover, as pushed by Krueger and Perri (2003a) as the important endogenous mechanism, and discussed by Heathcote et al. (2003), developments in the financial sector might have a strong implication on the consumption inequality.

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Appendix A: Estimation of the Earnings Process

This appendix gives details of the details of the estimation of the earnings process. Appendix A-1 gives detailed description of the data that are used for the estimation. Appendix A-2 gives the detailed description of the estimation procedure employed.

A-1 Data Description

We use the Panel Study on Income Dynamics (PSID). This is a longitudinal survey that follows a sample of households of the U.S. since 1968. Approximately 5000 households were interviewed in the initial year of the survey, and members of the originally-interviewed households and their offsprings have been interviewed annually, until 1997. After 1997, the survey is conducted bi-annually. The primary advantage of the PSID is its long coverage and the longitudinal nature. The primary disadvantage of the survey is that the sample size is relatively small.

We use 30 years of the sample periods between 1968 to 1997. This period is when the survey is conducted annually. The selection of sample years means that we have the data on earnings between 1967 to 1996, because the survey questions ask about the earnings for the *previous* year. The number of household/year observations is 155891. On average, there are 5196 observations each year. Following the standard in the literature, we include only the observations which satisfy all the 10 criteria below:

1. Head of household.
2. Male.
3. White.
4. Age between 20 to 59.
5. Positive earnings reported.
6. Earnings not top-coded.
7. Hourly wage implied by the earnings and hours worked above half of the minimum wage for each sample year.
8. Working more than 520 hours (8 hours a day, 5 days a week, for a quarter) in a year.
9. Working less than 5096 hours (14 hours a day, 7 days a week, for a year) in a year.
10. Exist in the sample after applying all the other criteria more than two consecutive periods.[9]

[9] Following the tradition of the literature using the PSID, households who drop out of the sample and reappear in the sample are treated as different households, in order to increase the sample size. There are not sufficiently large number of households who stay in the sample throughout all the survey years.

These criteria leave us with 48123 household/year observations, or 1604 observations on average per year. Table 9 summarizes the descriptive statistics out of the sample which satisfies the criteria. Average age is stable around 38. Average years of education shows an upward trend over the sample years, even though the data is top coded at 17 and thus underestimates the years of education for individuals with post-graduate education. The college premium, which is defined as the average earnings of individuals with college or higher degree relative the average earnings of the individuals with exactly high school diploma, shows an upward trend. The last column shows the proportion of individuals with at least college degree. The increase of the proportion together with the rising college premium is considered to be the evidence of the skill-biased technological change.

A-2 Estimation Procedure

The estimation procedure closely follows Heathcote et al. (2003). They pose a functional form for the time varying wage process and estimate the parameters of the wage process using the method developed by Chamberlain (1984). The essence of the method is to minimize the distance between the theoretical and sample covariance matrices. We employ the same methodology but estimating the time varying *earnings* process instead of the *wage* process.

We use the following functional form for the time varying earnings process for agent a of age i at time t, and estimate the parameters of the functional form using the PSID data described in the previous section.

$$\log(y_{i,a,t}) = \bar{y}_{i,t} + \hat{y}_{i,a,t} \tag{19}$$

$$\hat{y}_{i,a,t} = \pi_t \gamma_a + \eta_{i,a,t} + \phi_t \nu_{a,t} \tag{20}$$

$$\eta_{i,a,t} = \rho \eta_{i-1,a,t-1} + \tau_t \epsilon_{a,t} \tag{21}$$

The equation (19) states that the log earnings can be decomposed into the time-varying average life-cycle profile $\bar{y}_{i,t}$, and the shock components $\hat{y}_{i,a,t}$. The equation (20) states that the shock components $\hat{y}_{i,a,t}$ can be further decomposed into three components. The first component is the permanent shock $\pi_t \gamma_a$. It consists of the time varying loading factor π_t and the individual realization of the shock γ_a, which is drawn from $N(0,1)$. As in obvious from the specification, γ_a is drawn only one at birth. The second component is the persistent shock $\eta_{i,a,t}$. As the equation (21) states, $\eta_{i,a,t}$ follows an AR(1) process. The AR(1) process is characterized by the innovation term $\epsilon_{a,t}$ which is drawn from $N(0,1)$, the time-varying loading factor for the innovation term τ_t, and the persistence parameter ρ. The third component is the transitory shock $\phi_t \nu_{a,t}$. It consists of the time varying loading factor ϕ_t and the individual realization of the shock $\nu_{a,t}$, which is drawn from $N(0,1)$. We show below the procedure to estimate the time varying earnings process.

1. Regress the data to the following fourth order polynomial with age, which is the proxy for experiences, and zero mean residuals.

   $$\bar{y}_{i,t} = b_{0,t} + b_{1,t} i + b_{2,t} i^2 + b_{3,t} i^3 + b_{4,t} i^4 \tag{22}$$

   The residuals is defined as $\hat{y}_{i,a,t}$.

Table 9: Descriptive Statistics of the PSID Sample: 1967-1996

Year	Number of observations	Average age	Average education	Average earnings	Variance of log-earn	College premium	w/ College degree
1967	1398	39.1	11.6	42785.5	0.2653	1.593	0.162
1968	1570	39.1	11.9	43343.4	0.3110	1.432	0.204
1969	1544	38.6	12.1	44860.2	0.3030	1.429	0.213
1970	1541	38.6	12.2	44513.8	0.3270	1.466	0.218
1971	1583	38.4	12.3	44873.1	0.3216	1.411	0.226
1972	1621	38.2	12.4	46322.7	0.3345	1.403	0.231
1973	1649	37.9	12.5	46893.8	0.3385	1.343	0.238
1974	1647	37.8	12.6	45043.0	0.3403	1.401	0.246
1975	1638	37.5	12.7	43214.0	0.3605	1.406	0.249
1976	1644	37.5	12.8	45146.9	0.3449	1.401	0.247
1977	1662	37.4	12.8	45570.0	0.3344	1.432	0.244
1978	1675	37.5	12.8	46613.0	0.3318	1.407	0.247
1979	1692	37.6	12.8	45373.9	0.3345	1.396	0.248
1980	1682	37.7	12.9	43226.4	0.3481	1.455	0.251
1981	1672	37.6	12.9	42006.8	0.3657	1.484	0.254
1982	1663	37.7	13.0	42536.4	0.4444	1.647	0.263
1983	1649	37.7	13.1	43371.0	0.4308	1.663	0.266
1984	1697	37.7	13.5	45960.5	0.4397	1.687	0.303
1985	1695	37.8	13.5	46102.8	0.4925	1.772	0.301
1986	1687	37.8	13.5	46255.9	0.4792	1.821	0.302
1987	1680	37.7	13.5	45261.8	0.4613	1.759	0.303
1988	1674	37.8	13.6	46146.6	0.4709	1.783	0.309
1989	1671	37.9	13.6	46464.5	0.4581	1.926	0.309
1990	1648	38.0	13.6	46025.1	0.4934	1.987	0.307
1991	1634	38.1	13.6	46050.3	0.4617	1.931	0.308
1992	1550	38.3	13.7	48198.7	0.4812	1.904	0.312
1993	1471	38.2	13.6	49292.2	0.4906	2.073	0.311
1994	1528	38.5	13.6	49395.2	0.4873	1.965	0.312
1995	1527	38.8	13.6	49296.3	0.4943	2.011	0.316
1996	1131	39.7	13.9	52501.7	0.4422	2.052	0.354
total	48123	38.0	13.0	45668.4	0.3997	1.644	0.268

[1] Source: PSID, 1968-1997 waves.

[2] Earnings is denominated in 2000 U.S. dollars, converted using CPI.

[3] Education is defined as the number of years of education completed. It is top-coded at 17.

[4] College premium is defined as the ratio of average earnings of individuals who completed at least college degree to the average earnings of individuals with high school degree.

2. If we fix the parameters which characterize the shock component of the earnings process, $\{\pi_t\}$, $\{\tau_t\}$, ρ, $\{\phi_t\}$, we can compute the theoretical variances and covariances of $\hat{y}_{i,a,t}$. Specifically, the cross sectional variance at year t takes the following form:

$$var(\hat{y}_{i,a,t}) = \pi_t^2 + var(\eta_{i,a,t}) + \phi_t^2 \tag{23}$$

where

$$var(\eta_{1,a,t}) = \tau_t^2 \tag{24}$$

$$var(\eta_{i,a,1}) = \sum_{j=0}^{i-1} \rho^{2j} \tau_1^2 \tag{25}$$

$$var(\eta_{i,a,t}) = \rho^2 var(\eta_{i-1,a,t-1}) + \tau_t^2 \tag{26}$$

Notice that the equation (24) implies that the distribution of $\eta_{i,a,t}$ at birth is zero, and the equation (25) implies that the variance of the innovation term before period 1 is the same as in period 1. In other words, it is assumed that the process of $\eta_{i,a,t}$ is stable before period 1.

3. The theoretical cross-sectional covariance between period t and period $t-n$ (notice it has to be the case that $t > n$ and $i > n$) can be expressed as follows:

$$cov(\hat{y}_{i,a,t}, \hat{y}_{i-n,a,t-n}) = \pi_t \pi_{t-n} + \rho^n var(\eta_{i-n,a,t-n}) \tag{27}$$

4. Let us denote the theoretical variance-covariance matrix as Ω, and the vectorized one as $vec(\Omega)$.

5. We can also compute the sample counterpart of Ω. Let us denote it as $\hat{\Omega}$. In order to increase the sample size, we redefine the definition of age. In particular, it is assumed that age i contains data of individuals of actual age between $i-4$ to $i+4$. Since our original sample contains individuals of age between 20 and 59, under our new definition of age, the sample contains individuals of age between 25 to 55. The total number of age is now 31, instead of 40.

6. We obtain our estimates for the parameters such that the distance between Ω and $\hat{\Omega}$ is minimized. More precisely, if we denote the vector of the parameters as Θ, we are solving the following minimization problem, given a choice of a weighting matrix W.

$$\min_{\Theta} [vec(\Omega) - vec(\hat{\Omega})]^T W [vec(\Omega) - vec(\hat{\Omega})] \tag{28}$$

7. Regarding the choice of W, the identity matrix is used, following Heathcote et al. (2003) and other literature that employs the same estimation procedure.[10]

[10] These literature follows the finding by Altonji and Segal (1996). They show that for many common applications it is superior to use the identity matrix rather than to use the optimal weighting matrix characterized by Chamberlain (1984).

Appendix B: Calibration

This appendix gives detailed information associated with the calibration procedure.

B-1 Aggregate Statistics

This section describes the details of the procedure to construct the calibration targets of the baseline model. The baseline calibration is intended to replicate the U.S. economy is 1967, but with one important modification. The top 1% richest of households, ranked by the total wealth holding, are excluded. Therefore, the targets are expected to capture the aggregate behavior of the poor 99% of the households, whose earnings instability is estimated using PSID data. In this section, I will describe how to construct the original (including the top 1% richest) aggregate statistics, as well as the modified (excluding the top 1% richest) aggregate statistics, and how to use the aggregate statistics to pin down some of the parameters of the model.

The output of the model corresponds to the GNP of the U.S. in 1967, minus the housing consumption. Both data are compiled by the Bureau of Economic Analysis (BEA). Housing services consumption is excluded because it is not a part of the output in the model. The non-hosing GNP can be separated into the capital income and the labor income. As for the ambiguous components of income in the data, I follow the suggestion of Cooley and Prescott (1995) and distribute the ambiguous income into the two income categories using the ratio of the unambiguous components. This procedure gives the labor share and the capital share in the second and the third rows of Table 10.

Table 10: U.S. Aggregate Statistics in 1967

Statistics	Including top 1%	Excluding top 1%
Non-housing Output (Normalization)	1.0000	1.0000
Capital income (avg: 1967-1996)	0.2677	0.1767
Labor income (avg: 1967-1996)	0.7323	0.8233
Total capital stock (1967)	3.2651	2.5711
Non-housing capital stock (1967)	2.2842	1.5080
Housing capital stock (1967)	0.9809	1.0631
Non-housing depreciation rate (avg: 1967-1996)	0.0708	0.0708
Housing depreciation rate (avg: 1967-1996)	0.0161	0.0161
Non-housing net return (avg: 1967-1996)	0.0402	0.0464
Housing net return (avg: 1967-1996)	0.0789	0.0789

[1] Source: BEA and author's calculation.

The housing capital stock is the residential fixed asset (including both public and private), compiled by BEA. The non-housing capital stock is the sum of the three components below, all of which are obtained from BEA as well:

1. Stock of the fixed non-residential asset, including both private and public.
2. Stock of consumer durable goods.
3. Stock of inventories.

The first two are obtained from Fixed Asset Tables compiled by the Bureau of Economic Analysis (BEA), and the stock of inventories is constructed by combining the incomplete data of the stock of inventories and the data on the flow of inventories. The data on the capital stock in Table 10 are normalized by the non-housing GNP obtained above. The sum of the values of housing and non-housing capital stock divided by the non-housing GNP in 1967 is 3.27, which is within the range of the value of capital output ratio used in the literature.

The average (1967-1996) depreciation ratio of capital stock, computed by taking the ratio of the capital consumption and the capital stock, is 7.1% for non-housing capital, and 1.6% for housing capital. The average (1967-1996) rate of return of capital is 4.0% for non-housing capital and 7.9% for housing capital.

The next step is to modify the aggregate statistics that are constructed above to reflect only those associated with the bottom 99% of the wealth distribution. First of all, I use 1989 wave of the Survey of Consumer Finances (SCF) to compute the wealth inequality statistics. Table 11 exhibits the inequality statistics of the U.S. in 1989. SCF is a cross-sectional household survey whose focus is the asset portfolio of households. Because of the high concentration of many kinds of assets on the richest households, the survey over-samples the rich households, which are chosen based on the tax information. Therefore, SCF is regarded as the most reliable data source for the asset holding of the rich households in the U.S.

Table 11: U.S. Inequality of Income and Wealth in 1989

	Quintiles					Top		Gini
	1st	2nd	3rd	4th	5th	5%	1%	
Total wealth held	-0.002	0.012	0.052	0.130	0.807	0.543	0.299	0.789
Housing asset held	0.007	0.038	0.143	0.266	0.546	0.221	0.065	0.557
Non-housing asset held	-0.006	-0.002	0.001	0.054	0.953	0.722	0.430	0.918
Income earned	0.068	0.108	0.148	0.195	0.480	0.262	0.137	0.404

[1] Source: Survey of Consumer Finance, 1989 wave.
[2] Total wealth is used to rank households' richness.

As is well known, the wealth is extremely concentrated (Budría, Díaz-Gimenez, Quadrini and Ríos-Rull (2002)). In addition, as pointed out by Luengo-Prado (forthcoming) using durable wealth instead of housing wealth, the financial (non-housing) wealth is substantially more concentrated among the richest compared with the housing asset. In particular, the richest 1% of the U.S. households hold 30% of the total wealth, 7% of the housing asset, 43% of the financial asset, and 14% of the total income.

The last column of Table 10 exhibits the aggregate statistics if we exclude the richest 1% of the households. Non-housing capital stock relative to non-housing output declined from 2.28 to 1.58, because the richest 1% holds 43% of the non-housing asset, while they earn only 14% of the total income. In other words, adjusted non-housing-capital-output ratio is computed by $\frac{2.28(1-0.43)}{1-0.14} = 1.53$. The housing asset relative to the non-housing output increased from 0.98 to 1.06, after the similar adjustment procedure as the non-housing capital. The ratio increased because the richest 1% only hold 7% of the housing asset, while they earn 14% of the total income. Consequently, the capital output ratio declines from 3.27 to 2.57.

Depreciation rate for both types of capital are unchanged. The capital share of income declined from 0.27 to 0.18, because the richest 1% holds substantial proportion of the non-housing asset and thus earns substantial proportion of capital income. The ratio of labor income earned by the richest 1% is relatively smaller compared with the ratio for the capital income. The net (of depreciation) rate of return of housing capital is unchanged, while the net (of depreciation) rate of return of non-housing capital is slightly adjusted (slightly increased from 4.0% to 4.6%) in order to make it consistent with the adjusted capital income ratio and adjusted capital output ratio.

Appendix C: Computation

The appendix describes how to compute a steady state equilibrium.

C.1 Data Structures

There are four individual state variables (i, z, h, k), where i is age, z is shock to earnings, h is housing asset holding, and k is non-housing asset holding. i and z are discrete. There are I possible values for i, and n_z possible values for z. h and k are defined over continuous spaces. h is discretized and can take one of n_h values. I put n_k grid points on the space of k.

The value function $V(i, z, h, k)$, and optimal decision rules $g_h(i, z, h, k)$ and $g_k(i, z, h, k)$ are stored using four dimensional arrays where the first, second, third, and fourth dimension have I, n_z, n_h, and n_k elements, respectively.

The choice of housing asset holding is restricted to the discretized space with n_h elements, while the choice of non-housing asset holding is not restricted on the grid points.[11] In order to evaluate

[11] Alternatively, it is possible to approximate the value function with two (h, k) variables, using two dimensional function. However, it turned out to be difficult to use two dimensional approximation without losing stability of

the value off the grid points with respect to k, shape-preserving spline approximation developed by Schumaker (1983) is used.

We need to find an invariant distribution of agents in order to compute aggregate demand of goods and assets and thus compute equilibrium prices. We approximate the type distribution of agents by a distribution of finite number of agents. An agent is characterized by its individual type (i, z, h, k), and we use the type distribution of large number of agents to approximate the type distribution.

C.2 Algorithm

The following algorithm is used to find a steady state equilibrium.

1. Guess a set of prices $\{p_h, p_k\}$.
2. Given $\{p_h, p_k\}$, solve the agents' optimal decision rules $g_h(i, z, h, k)$ and $g_k(i, z, h, k)$. Since the agents are finitely-lived, the optimal decision rules can be solved backwards, starting from the last period of life. In particular, the following method is used to solve the optimization problem.
   (a) For a type (i, z, h, k), the optimal choice of the housing asset holding h' is solved by grid search. In particular, values conditional on the choice of h' and the associated optimal choice of k' (derived using the algorithm below) are compared.
   (b) The optimal choice of financial asset holding, conditional on the choice of housing asset holding, is computed using Ridder's method, as implemented in Press, Teukolsky, Vetterling and Flannery (2001). Ridder's method is an extension of the false position method. The choice of the financial asset position is not restricted on the grid points. Values and derivative off the grid points are computed using shape-preserving spline approximation.
3. Using the optimal decisions that are just obtained, run a simulation with large number of agents. 100000 agents are used for each generation.
4. Calculate aggregate statistics, based on the simulation results. In particular, total demand for housing and financial assets are needed to check equilibrium conditions.
5. Check if market for housing and financial assets are cleared. If not, update the guess for the prices $\{p_h, p_k\}$ and start over. In particular, a price is lowered if there turns out to be an excess supply, and vice versa. However, the adjustment has to be slow because two prices are simultaneously adjusted, and the two assets are close substitutes.

the algorithm.

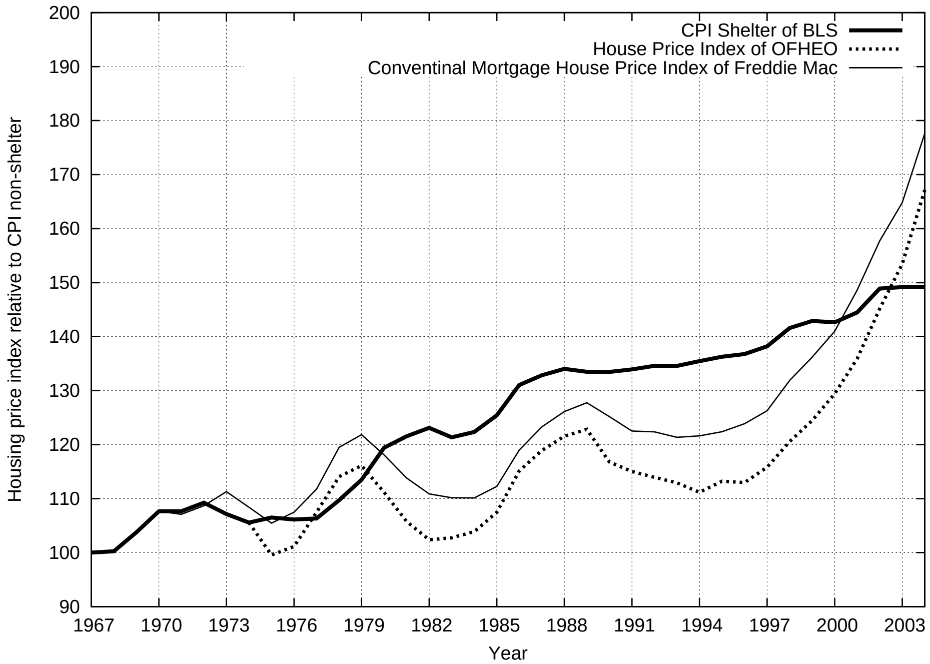

Figure 1: Housing price index in the U.S. during 1967-2004

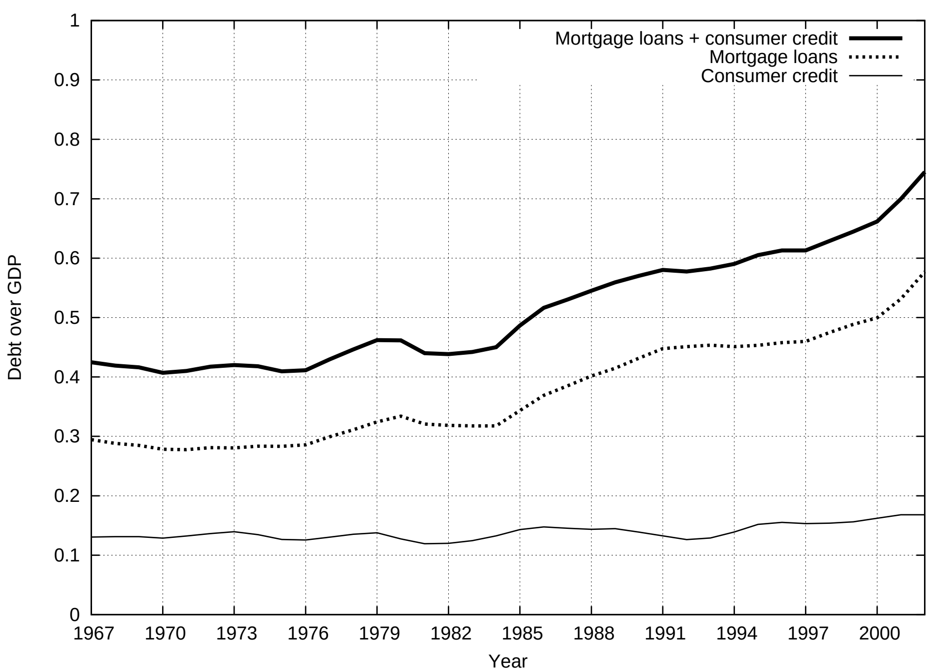

Figure 2: Debt over GDP in the U.S. during 1967-2002

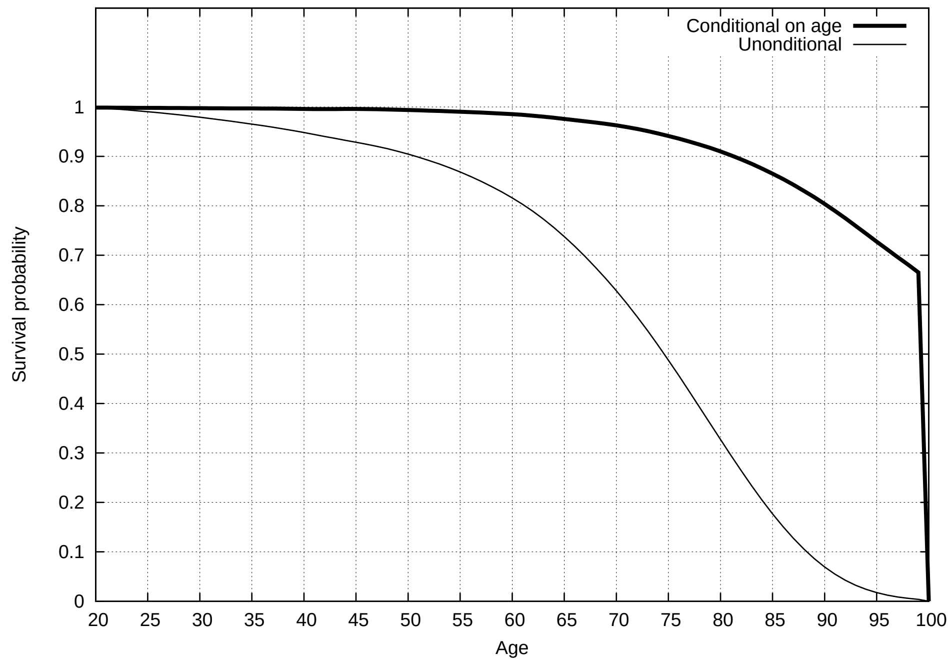

Figure 3: Survival probability

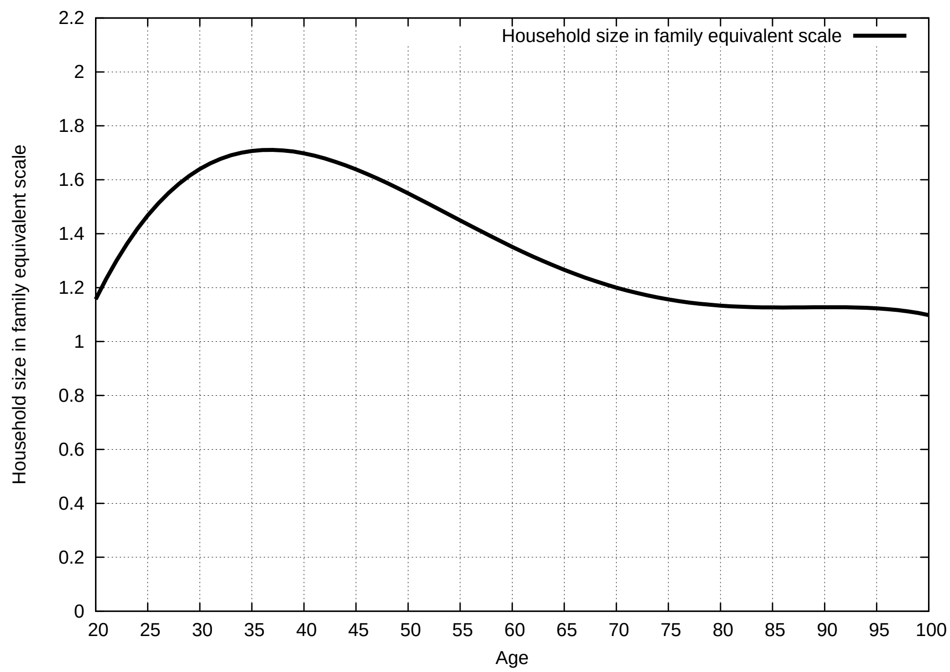

Figure 4: Household size in family equivalence scale

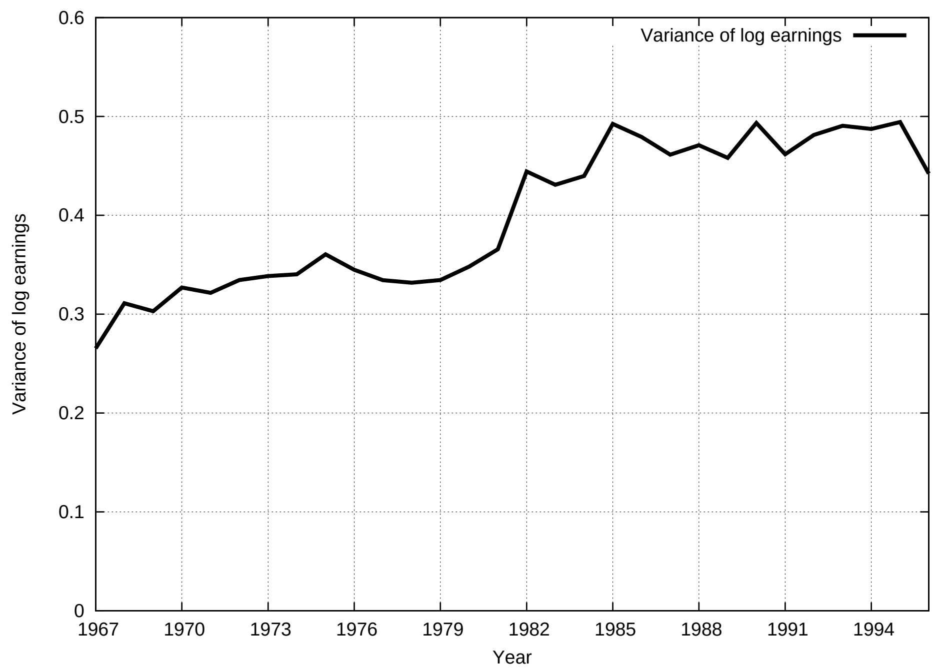

Figure 5: Variance of log earnings

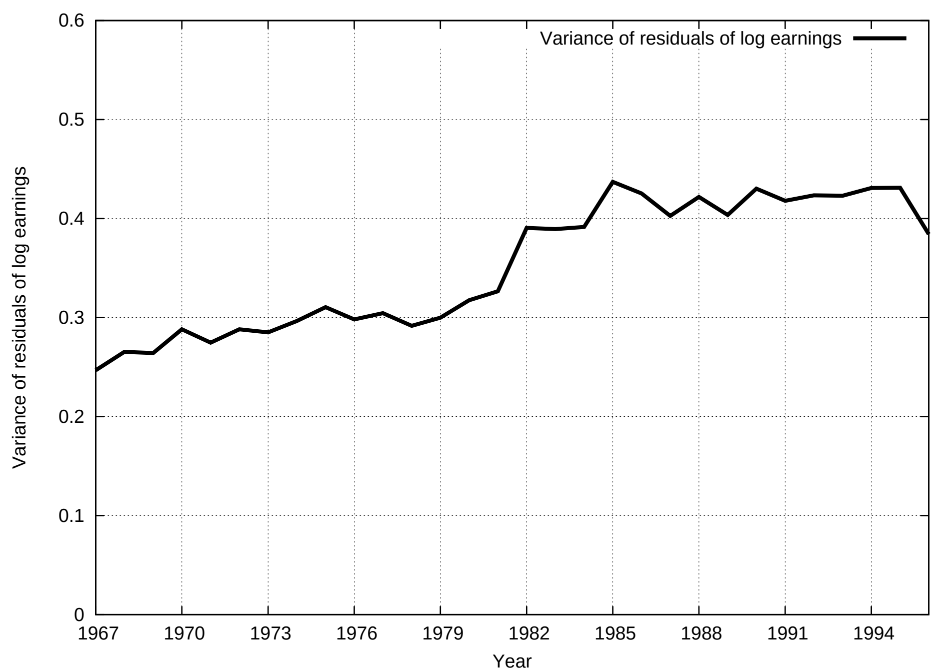

Figure 6: Variance of residuals of log-earnings regression

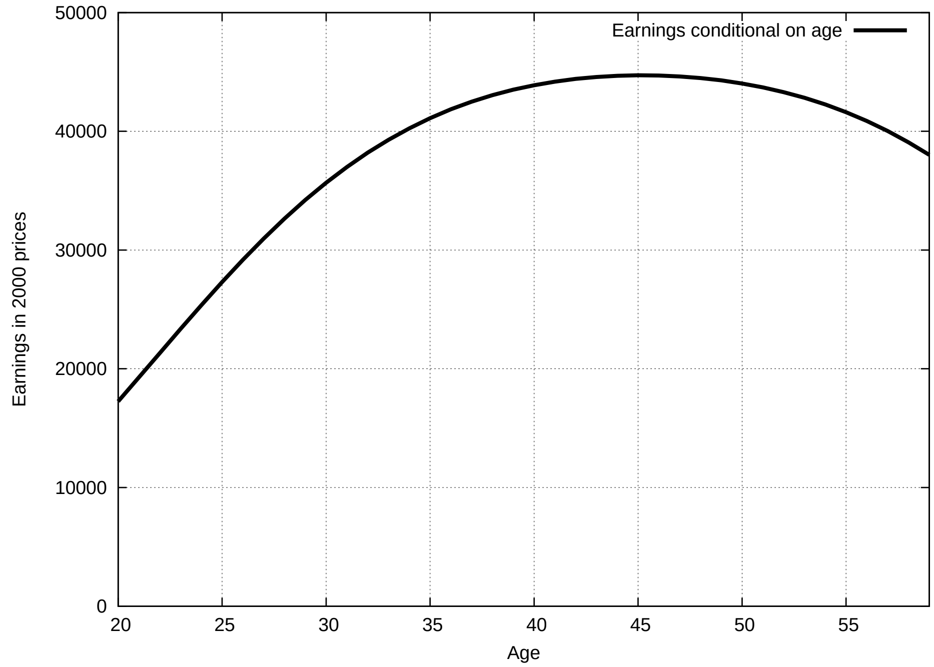

Figure 7: Average life-cycle earnings profile

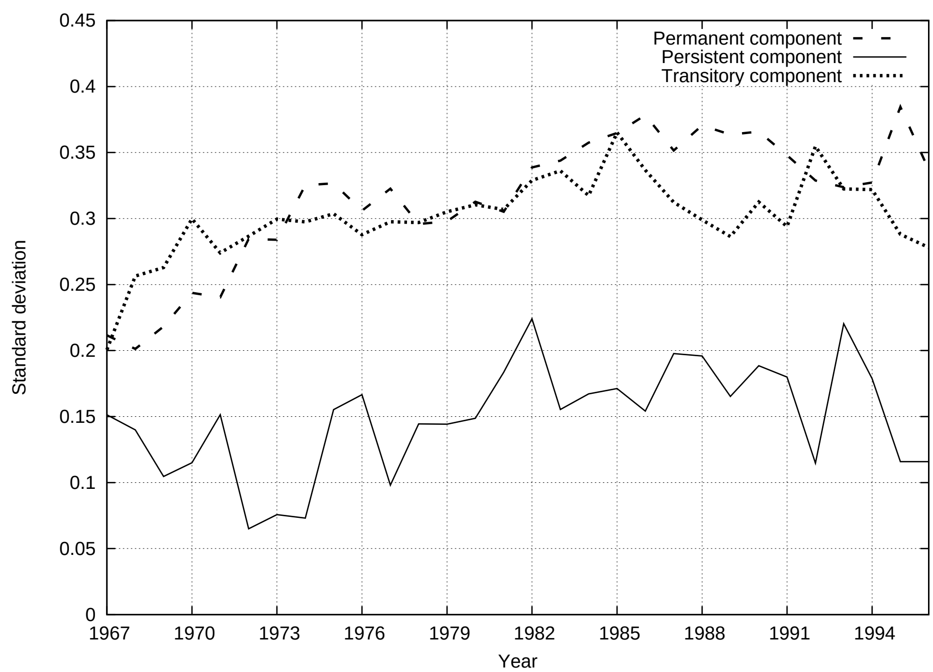

Figure 8: Changes in the standard deviation of shocks to earnings

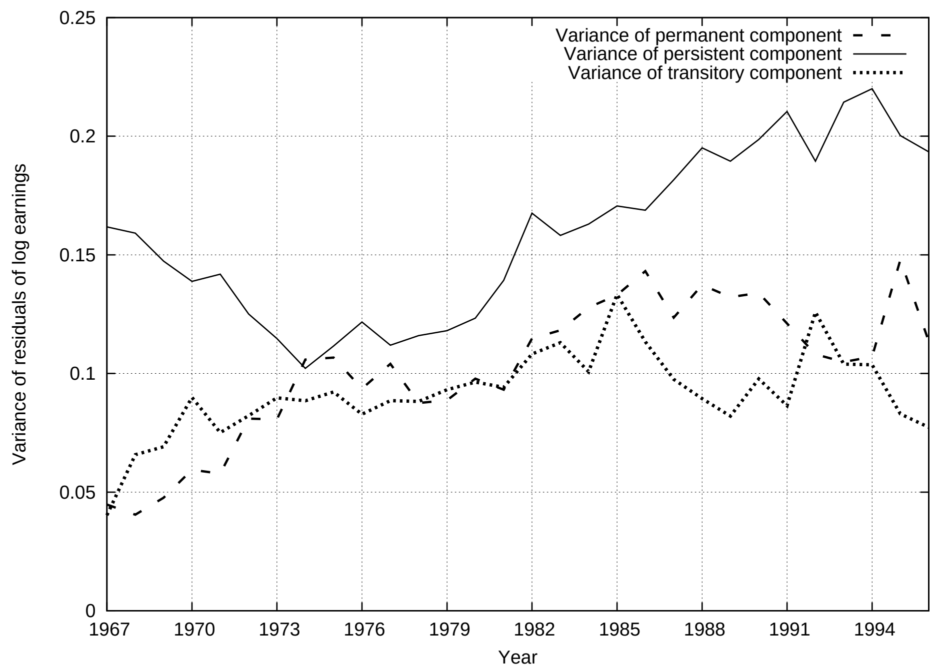

Figure 9: Decomposition of changes in variances of residuals of log-earnings

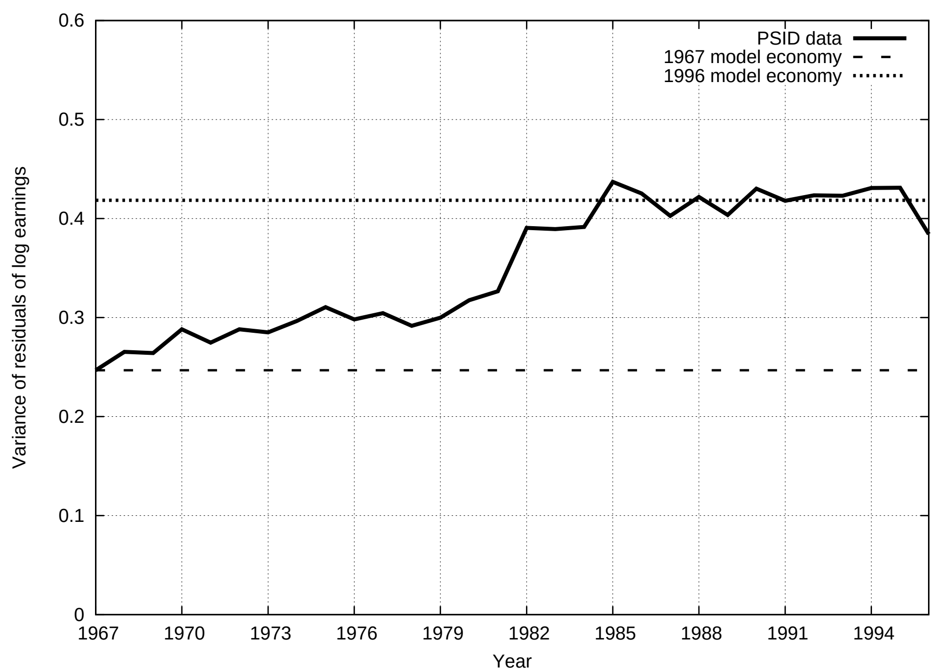

Figure 10: Comparison of the cross-sectional variances of the residuals of log earnings

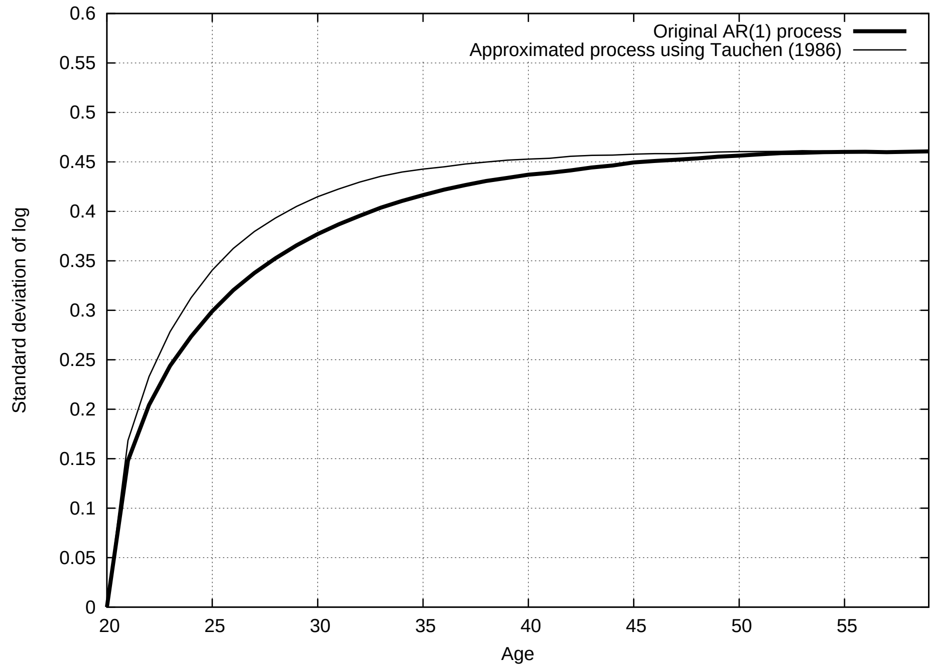

Figure 11: Comparison of the evolution of the cross-sectional variances between the original AR(1) process and the approximated process

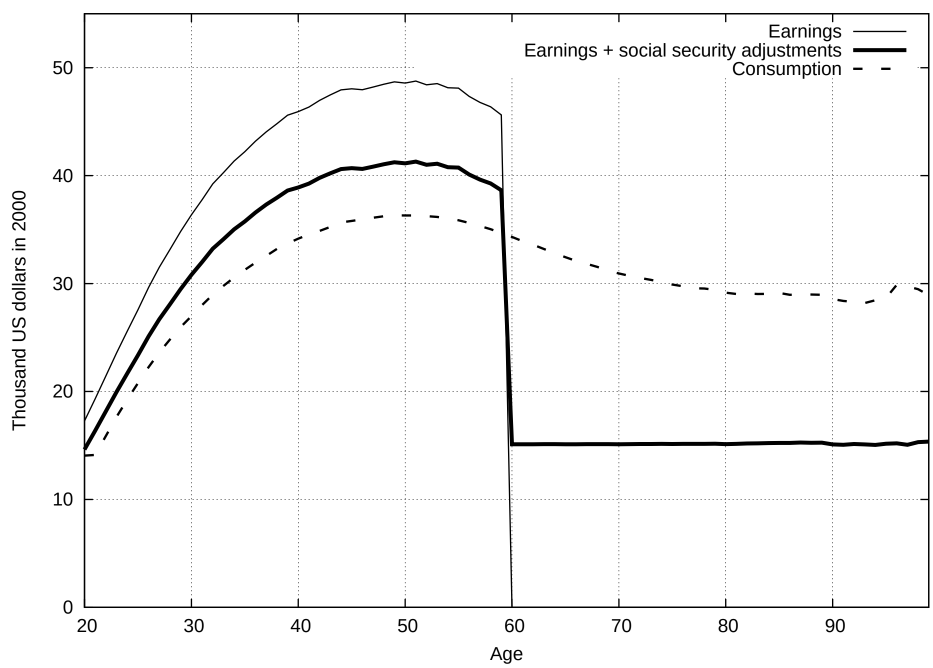

Figure 12: Average life-cycle profile of earnings (with adjustments associated with social security contribution and benefit) and consumption

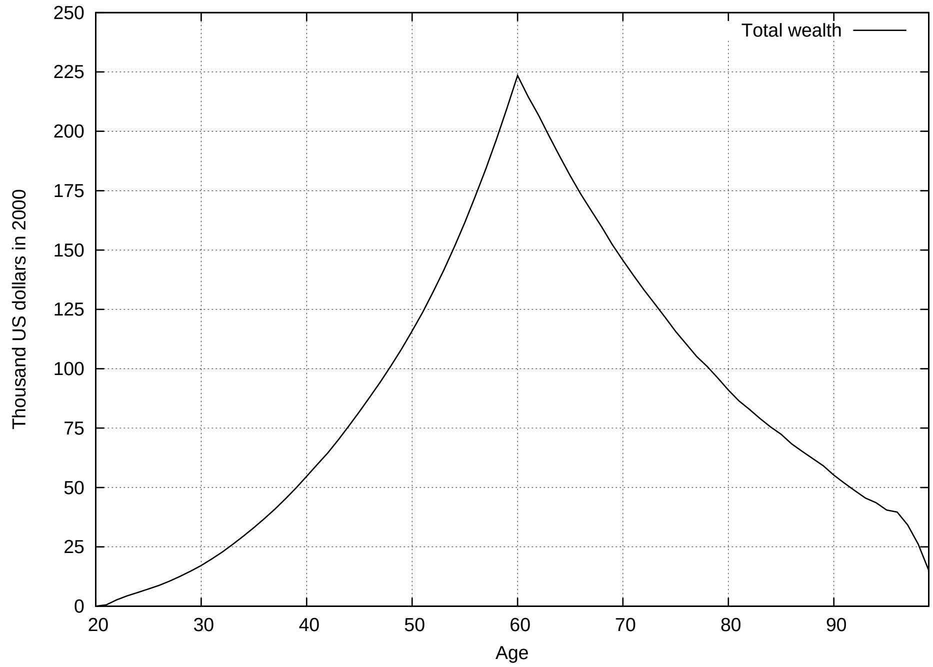

Figure 13: Average total wealth holding over the live-cycle

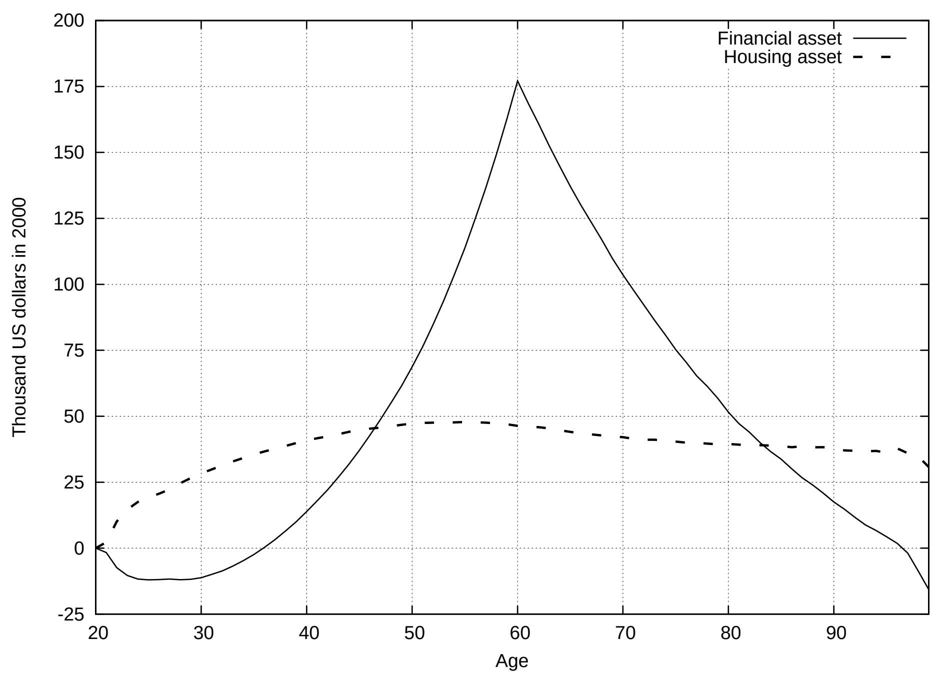

Figure 14: Average financial and housing asset holding over the life-cycle

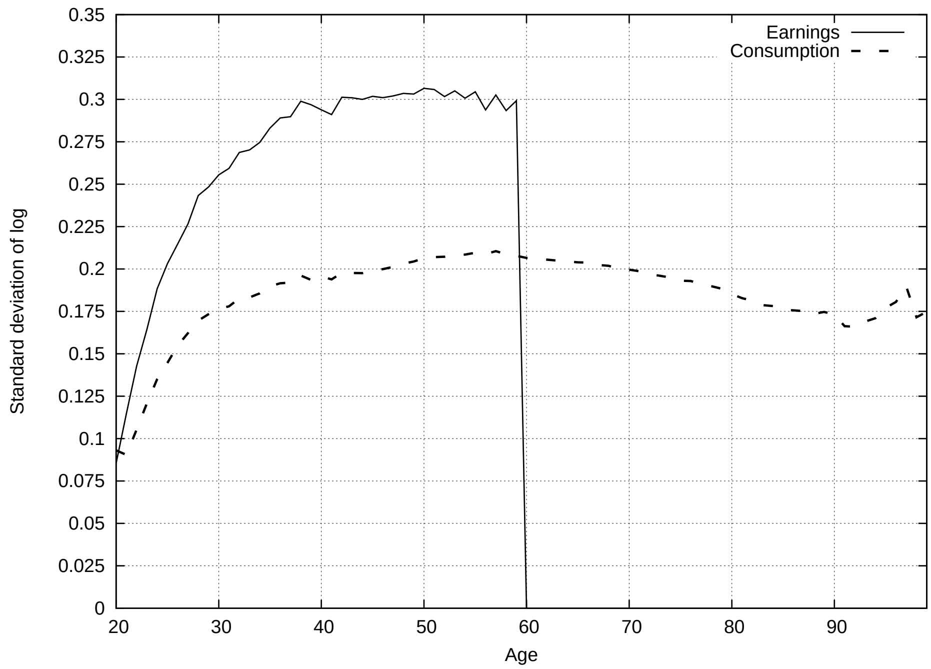

Figure 15: Cross-sectional variance of log of earnings and consumption

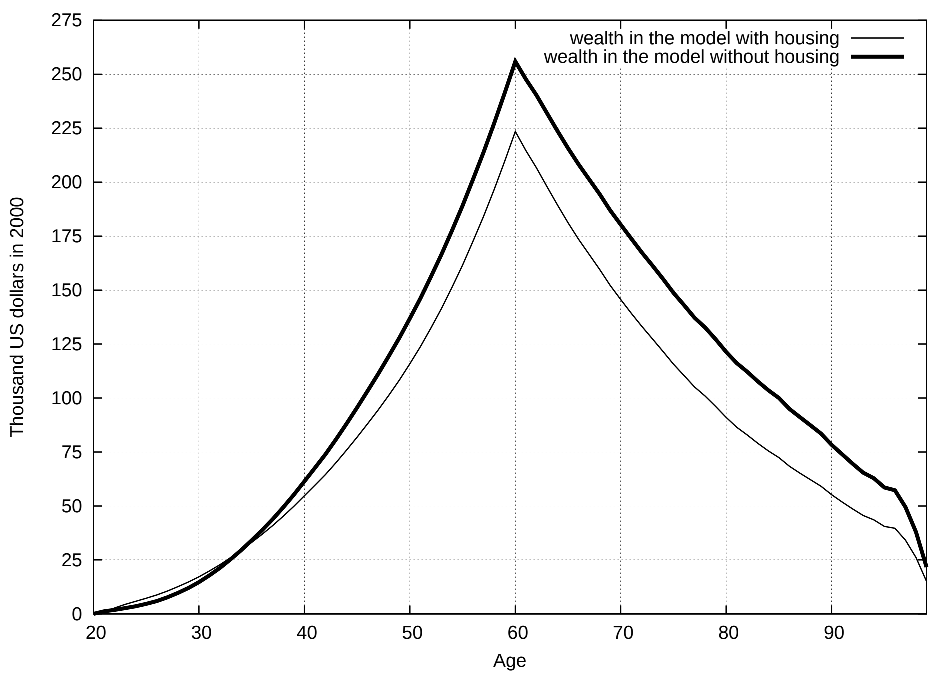

Figure 16: Life-cycle profile of total wealth

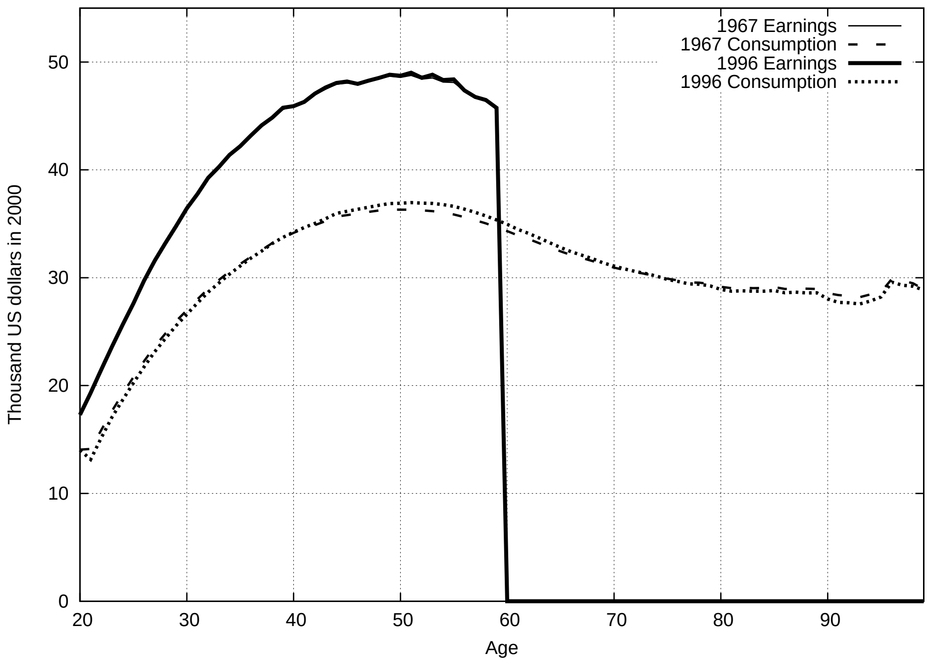

Figure 17: Changes in the average life-cycle profile of earnings and consumption

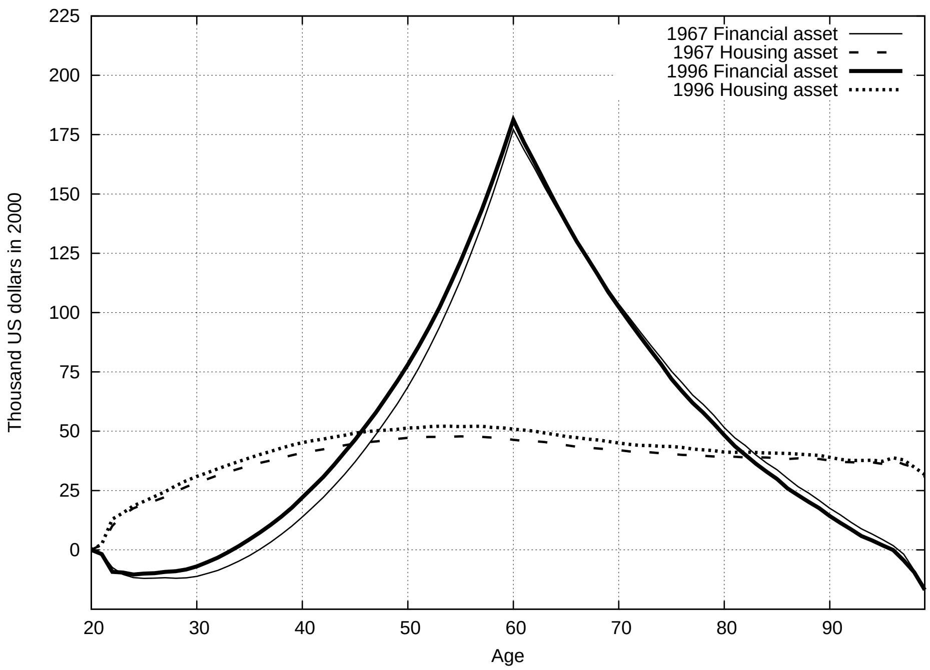

Figure 18: Changes in the average portfolio choice over the life-cycle

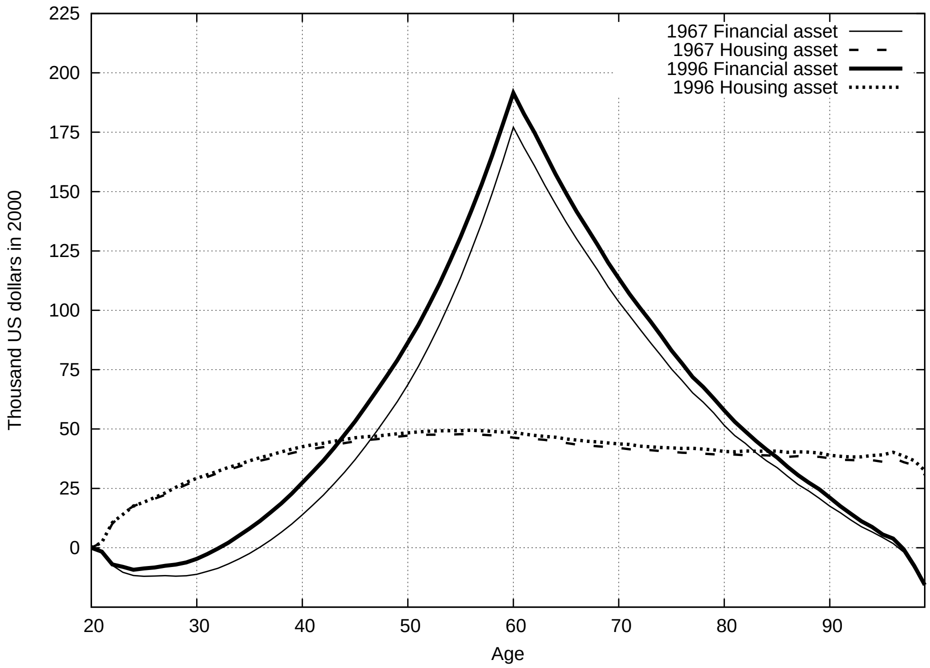

Figure 19: Changes in the average portfolio choice over the life-cycle, without general equilibrium effect

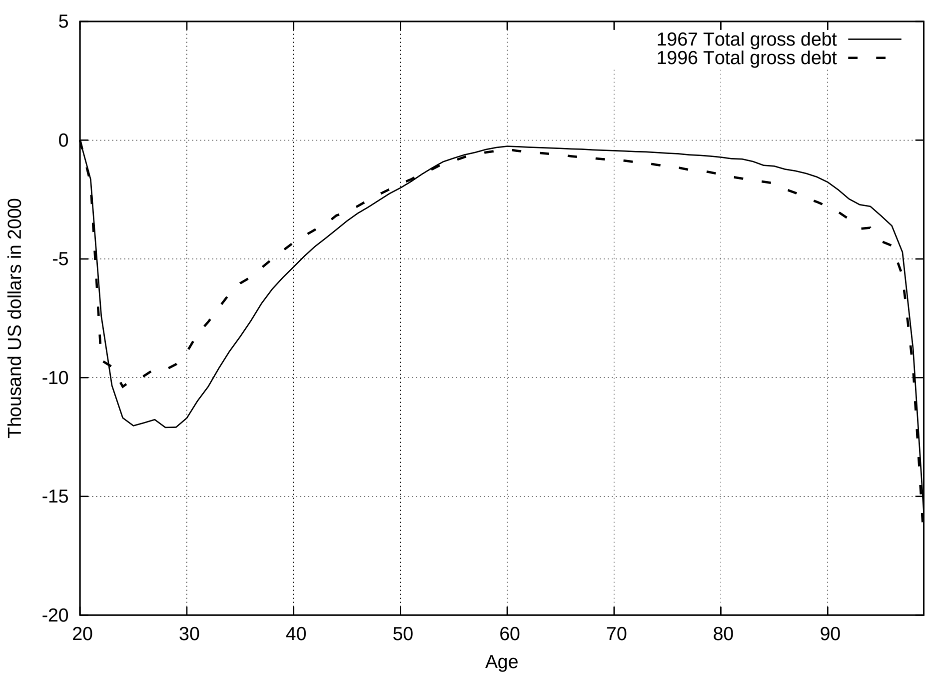

Figure 20: Changes in the amount of mortgage debt over the life-cycle